Status of the LSND and MiniBooNE anomalies after recent MicroBooNE results

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Vikram Discussion Meeting
PRL, Ahmedabad
Feb 19-21, 2025

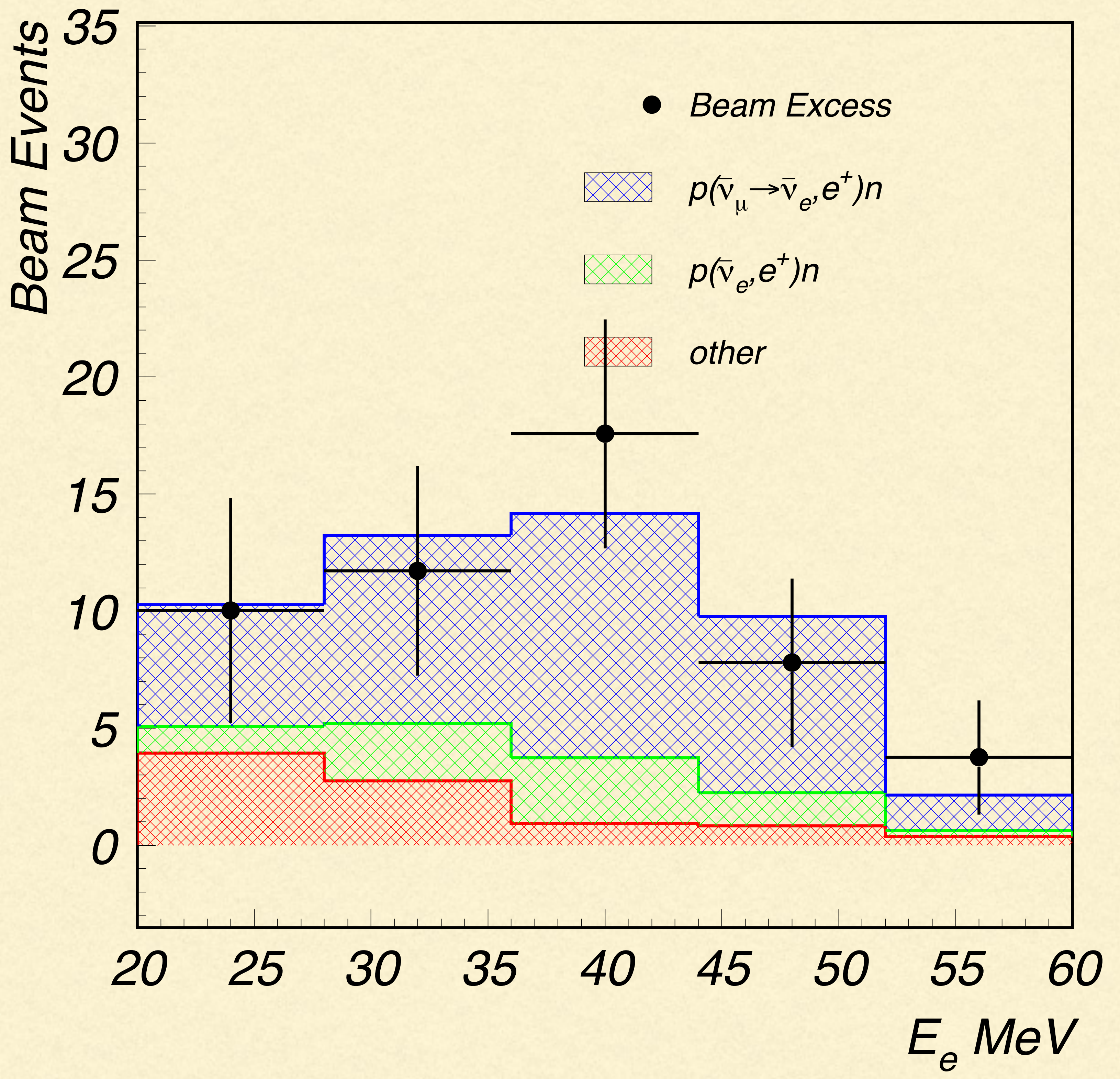
Why are they important?

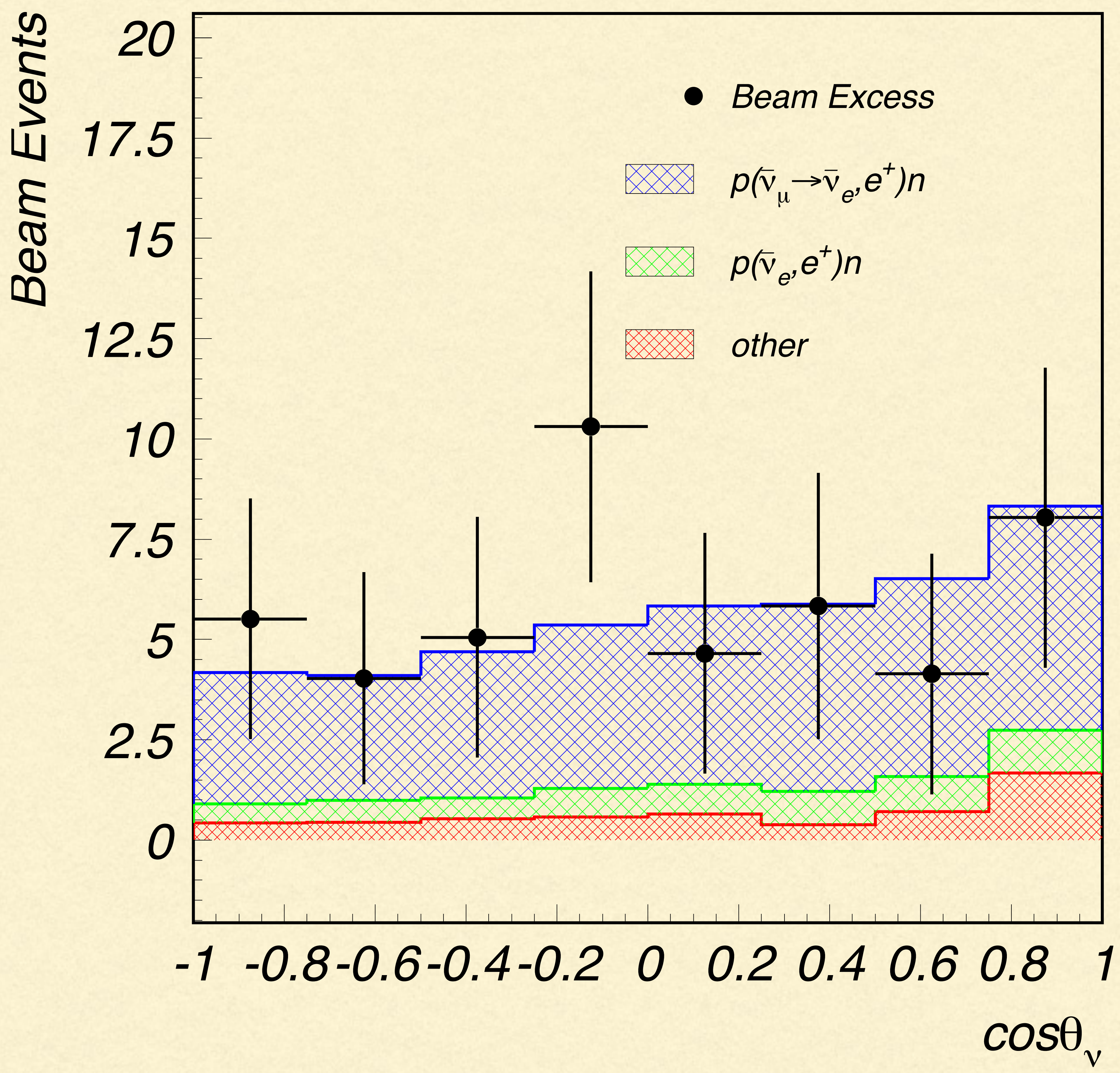
LSND and MiniBooNE are persistent and statistically significant anomalies which may have brought us to the cusp of new physics discoveries.

Extensive theoretical proposals have revealed that the new physics may involve new mediator(s) and interactions and a dark portal.

The latest set of MicroBooNE results (Feb 2025) constrain many of these proposals and it is thus germane to review the situation at this point in time

Anomalies at Short Baselines…….LSND (1993-1998)

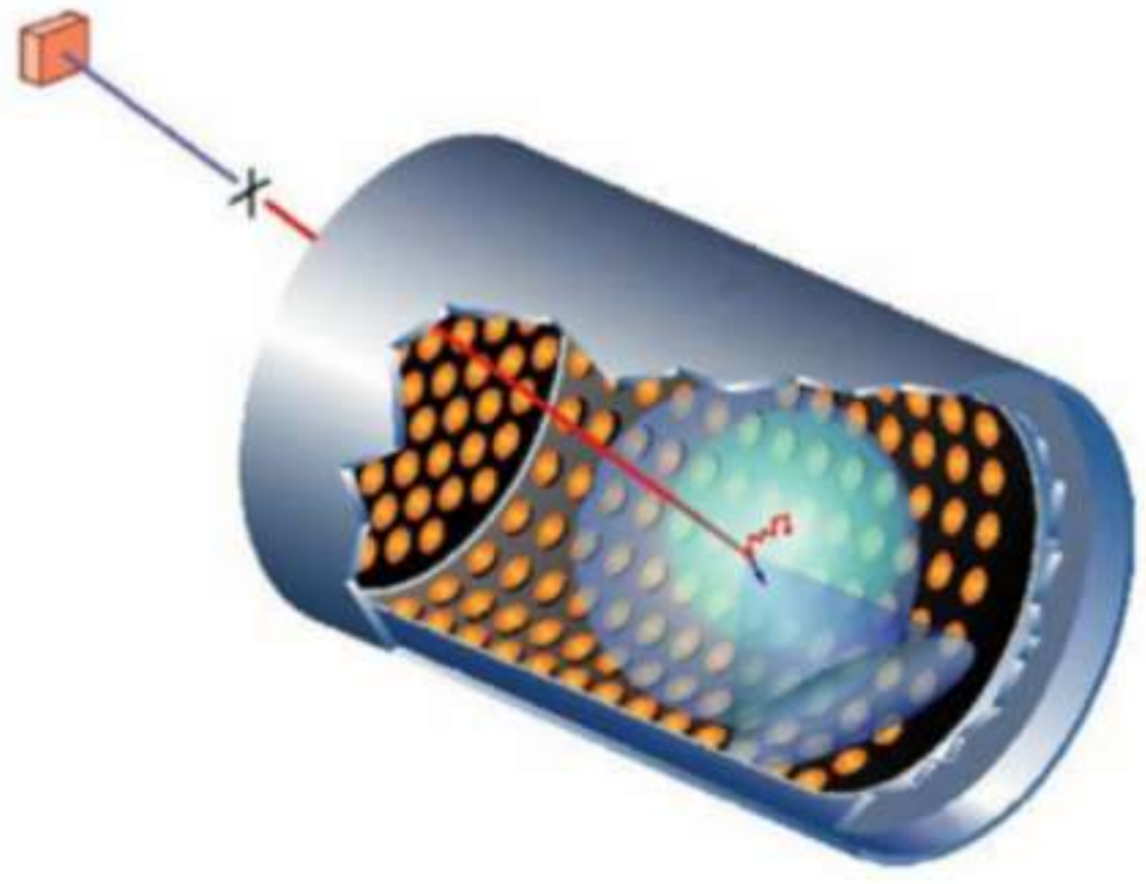




LSND

(3.8σ!)

Mineral oil scintillator detector



- Observation of unexplained electron-like excesses in the LSND at a level of 3.8σ above SM backgrounds.

Blue hatched region is oscillation fit.

- Note that unlike MB, both energy and angular distributions are relatively flat

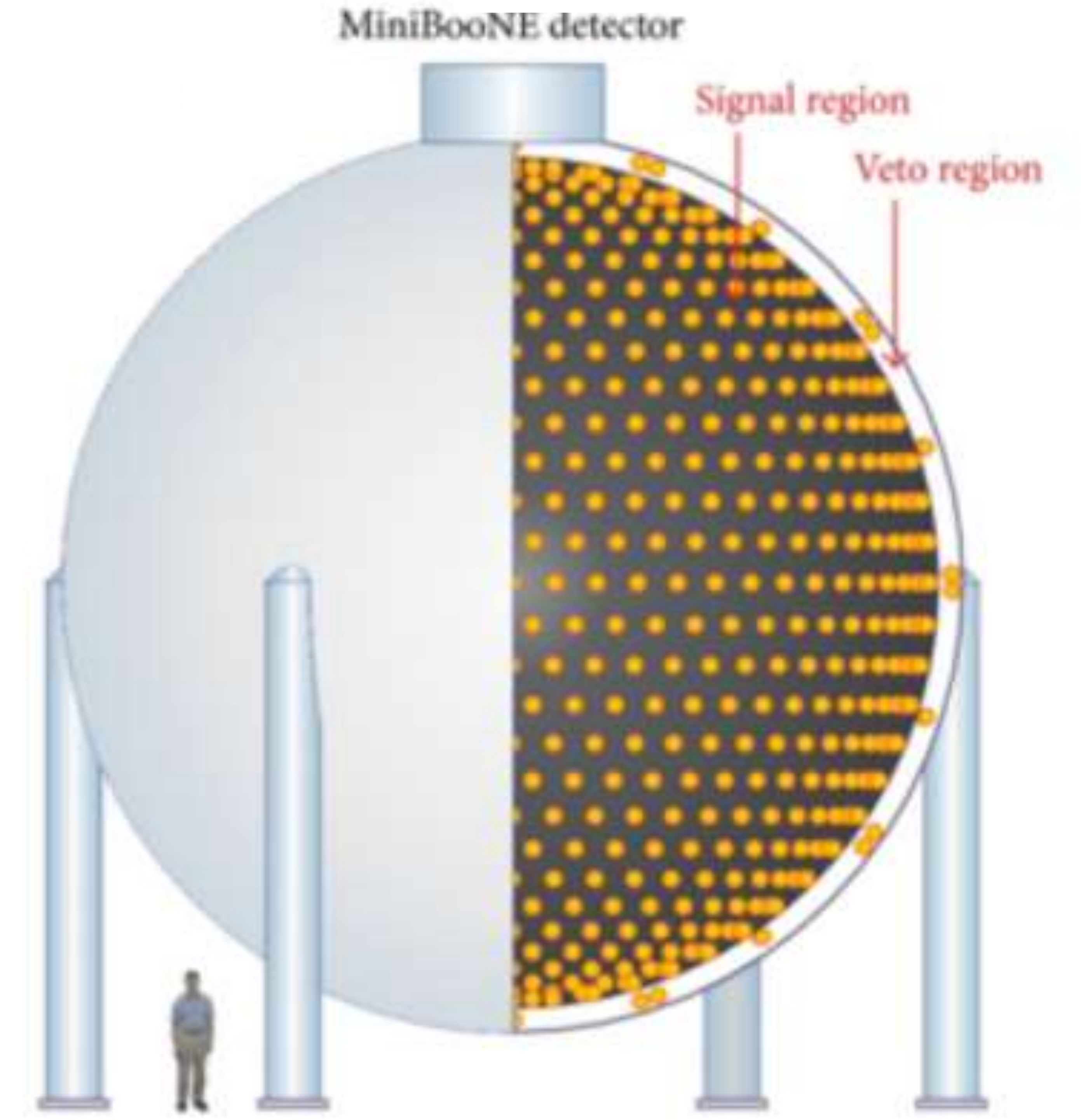
-

$\bar{\nu}_\mu$ From decay at rest (DAR) ν_μ From decay in flight (DIF)

Anomalies at Short Baselines…….MiniBooNE (2002-2017)

MiniBooNE

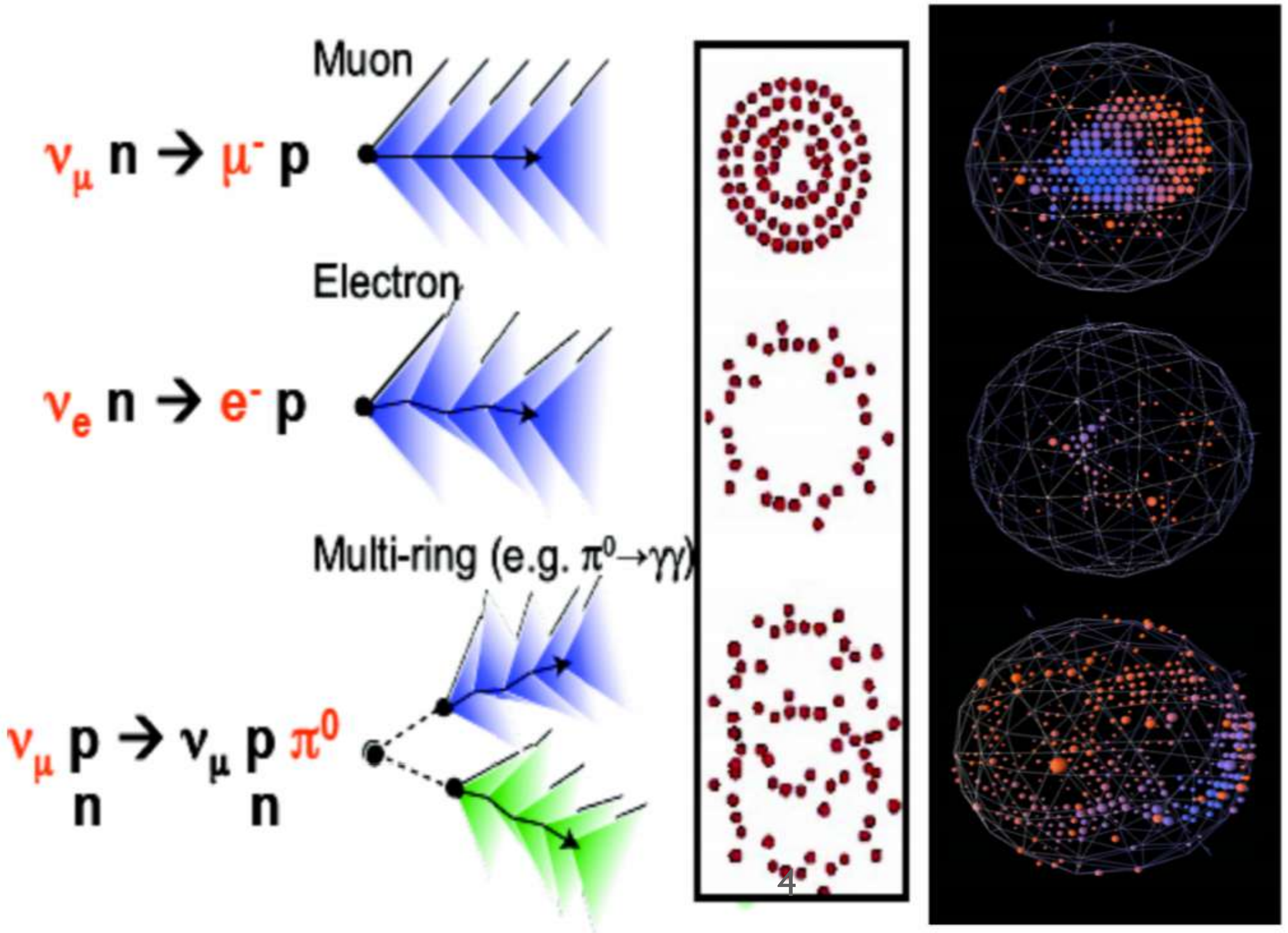
(4.8σ!)



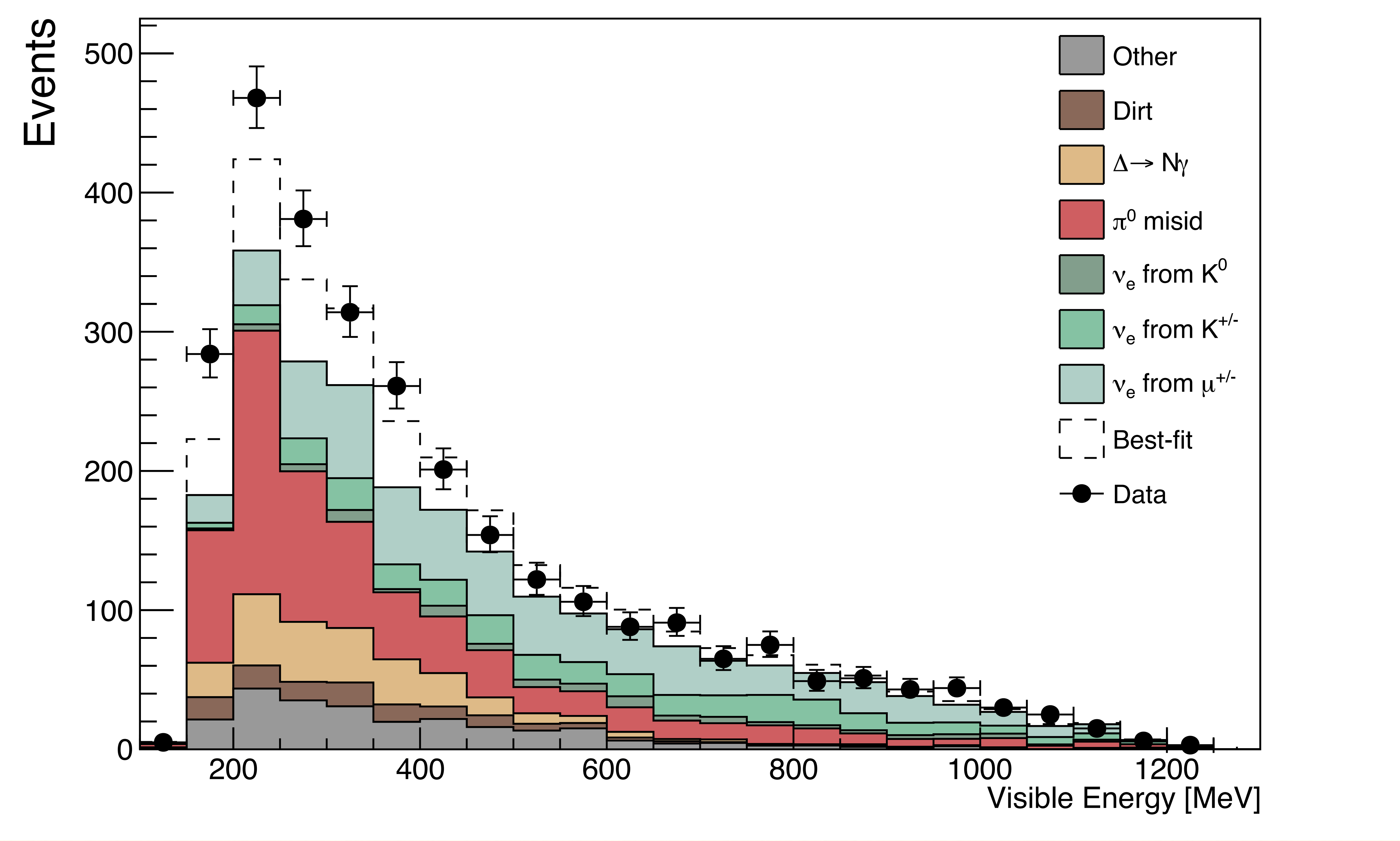
Mineral oil detector, 541 m baseline, 600 MeV (ν_μ) and 400 MeV ($\bar{\nu}_\mu$) peak fluxes.

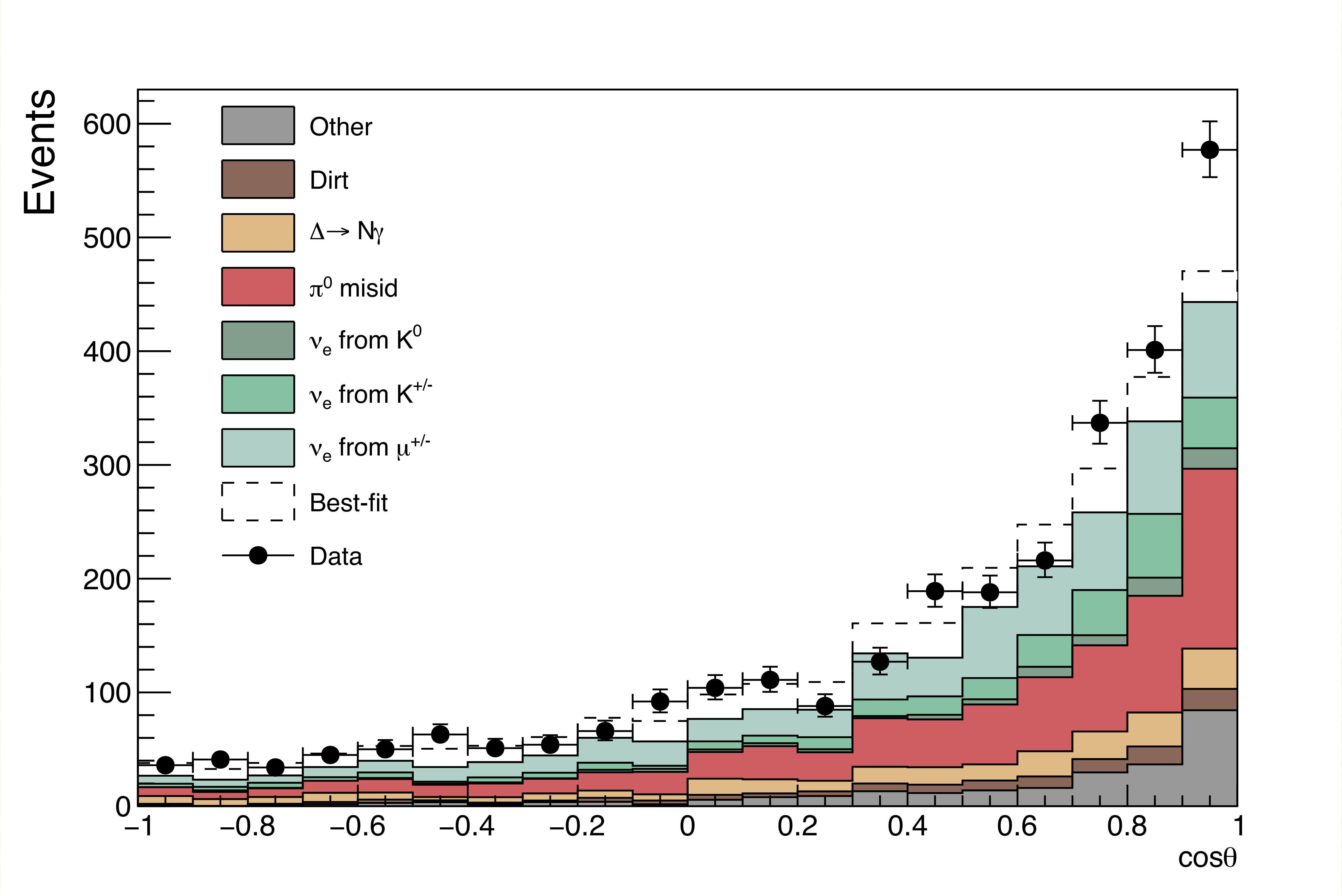
Was specifically built to test the LSND anomaly. Larger L, larger E, same L/E.

Three typical event signatures:
- Muon-neutrino CCQE produces sharp photon ring on PMTS,
- Electron-neutrino CCQE events produces fuzzy ring,
- Muon-neutrino NC can produce π_0: two gammas -> two fuzzy rings.



Anomalies at Short Baselines…….MiniBooNE





- A 4.8σ excess in electron-like events for neutrino and antineutrino modes in the MiniBooNE (MB) detector is observed

- SM: 2309 events
  Data: 2870
  Excess: 560

Excess is not small. Note it is at level of important SM backgrounds

Distinctive energy and angular distribution

Dashed line is sterile-active oscillation fit. Not a good fit at low energies or forward angles where most events present

MiniBooNE status……..

Possible systematics like :

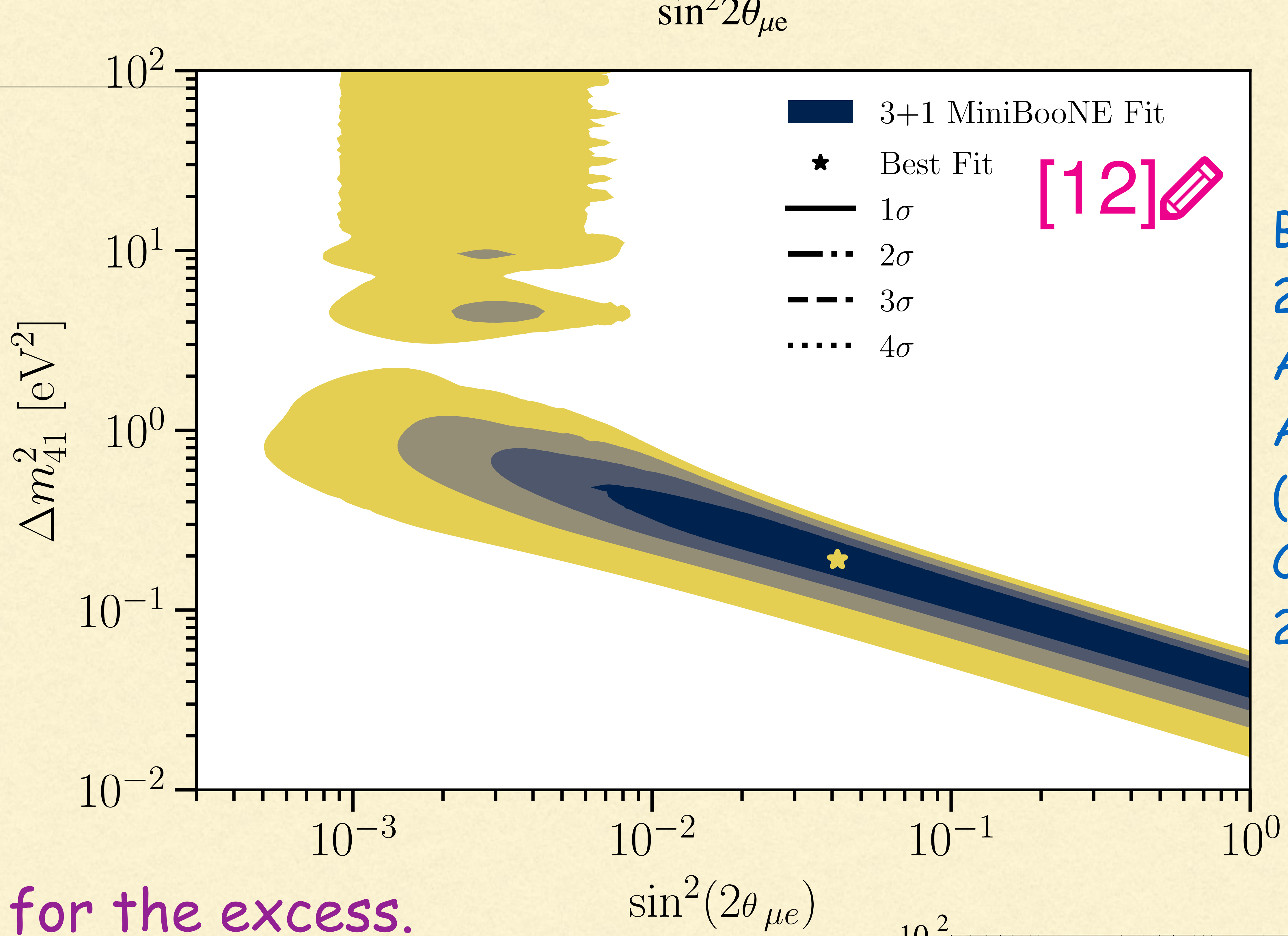
—Single photon from NC misidentified as
 e^- from νe

—π^0 photon identified as e

— incorrect reconstruction of neutrino energy

Have been extensively tested for .

At present, no combination of these can account for the excess.

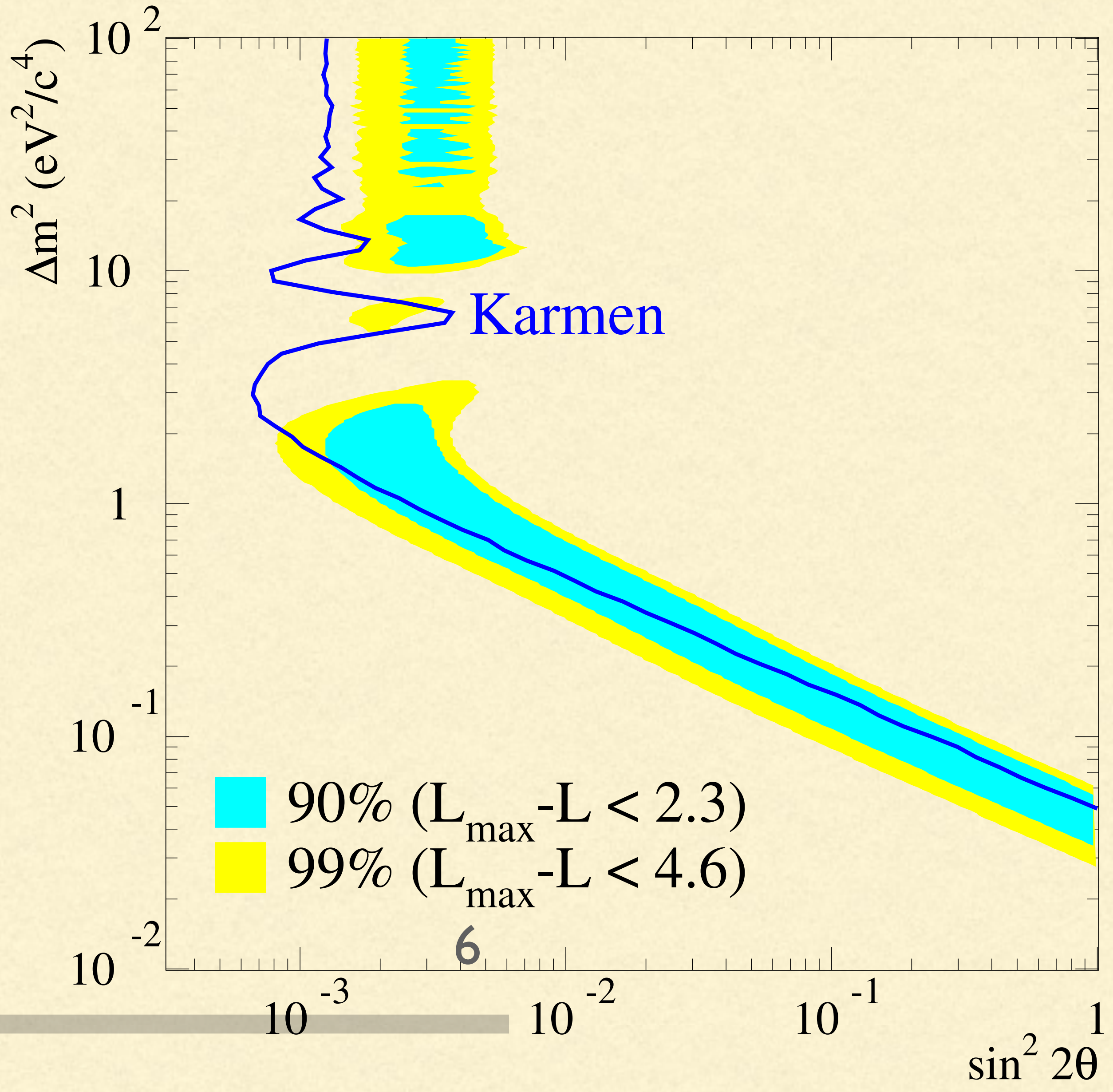


Brdar and Kopp,
2109.08157,
A.A.Aguilar-Arevalo et al
(MiniBooNE Collaboration)
2201.01724

Earlier oscillation allowed region for MB has been revised after

accounting for $\bar{\nu}e$ beam contamination and $\bar{\nu}_\mu$ calibration.

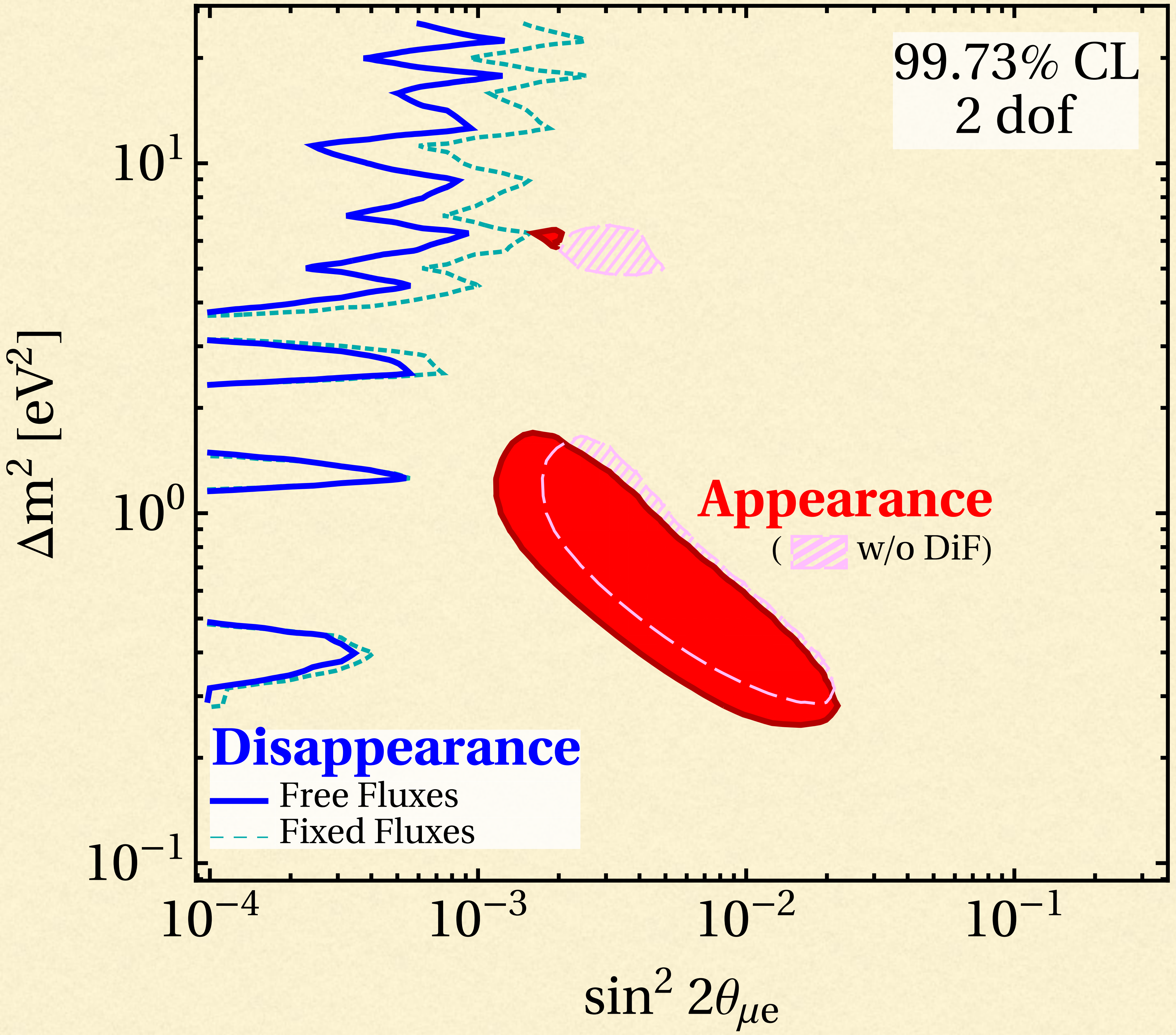
Note overlap with allowed LSND region



Maltoni, Nu 2024 talk

Tension between appearance and disappearance for active-sterile oscillations

- Combined analyses to test the active-sterile hypothesis for short baseline anomalies by various groups all reveal a common underlying problem: **Strong tension between appearance and disappearance data**



Dentler et al 1803.10661

Additionally, eV scale sterile neutrinos are constrained by Cosmology…….

Any relativistic neutrino species will contribute to the energy density of the Universe as radiation. Their total contribution may be parametrised by the parameter N_{eff}

Cosmology is sensitive to neutrinos in a way that is complementary to laboratory searches. It is less sensitive to individual masses and mixings, but is more directly affected by the absolute mass scale,

$$\frac{\rho_r - \rho_\gamma}{\rho_\nu^{\mathrm{std}}} = N_{\mathrm{eff}}\,,$$

e ρ_r is the total radiation energy density, ρ_γ is the photon contribution

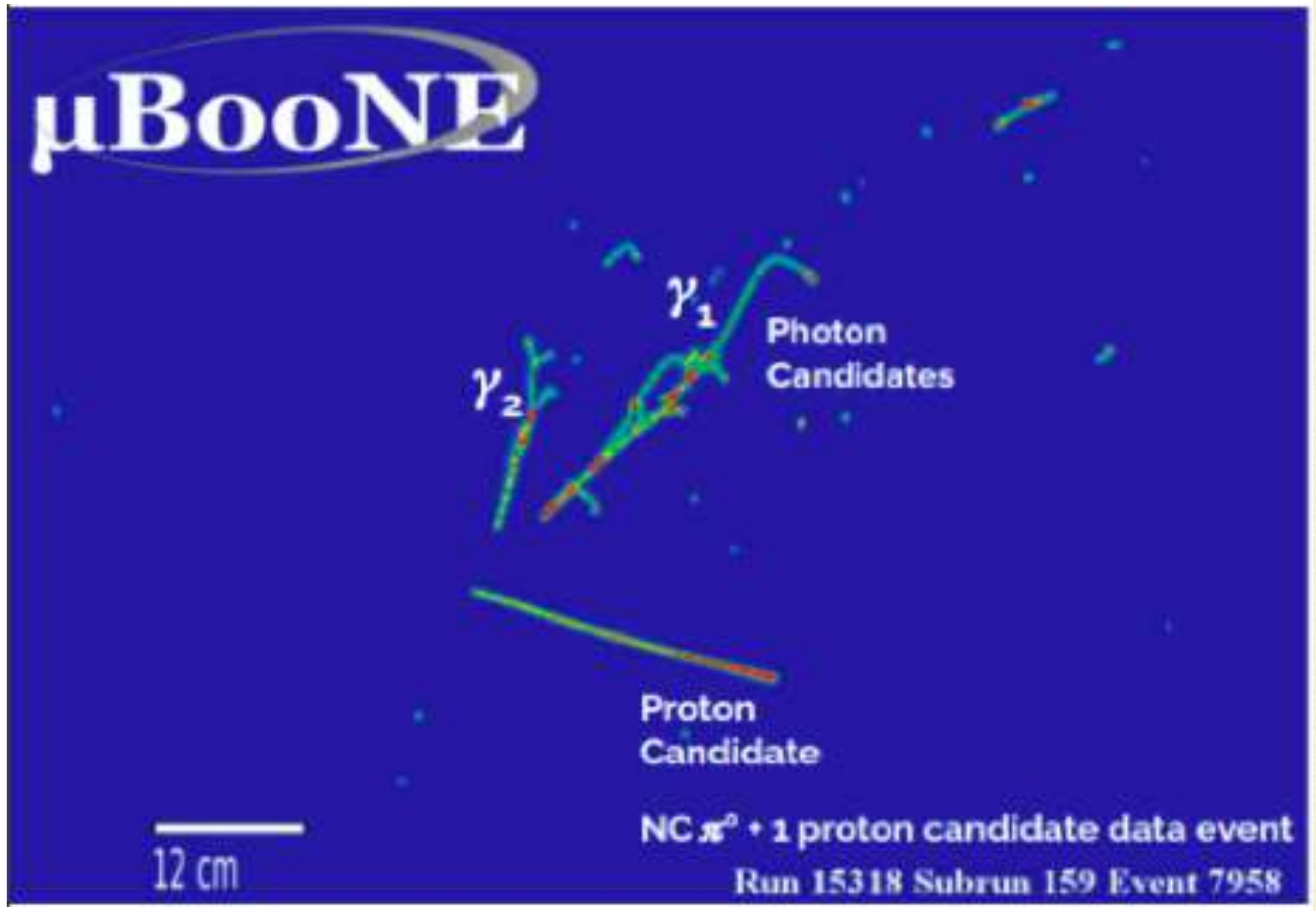
$$\rho_\nu^{\mathrm{std}} = 2 \times \frac{7}{8}\,\frac{\pi^2}{30}\left(\frac{4}{11}\right)^{4/3} T^4 .$$

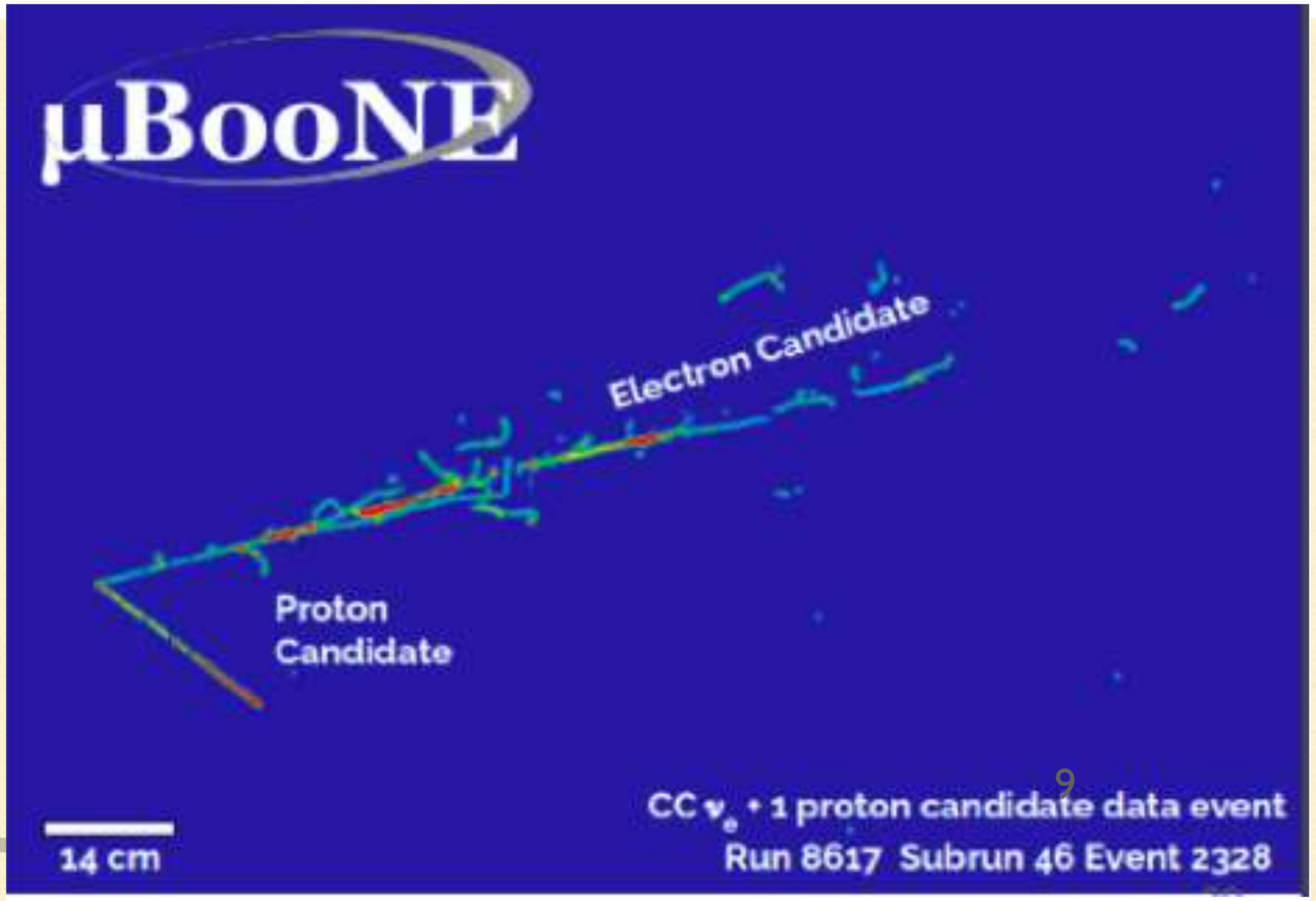
However, N_{eff} = 3.044 +- 0005 in the SM, leaving no space for an additional sterile relativistic neutrino species

Also, from PLANCK data,

$$\sum m_\nu < 0.26\,\mathrm{eV}\ (95\%\mathrm{CL}).$$

MicroBooNE (to test MB)



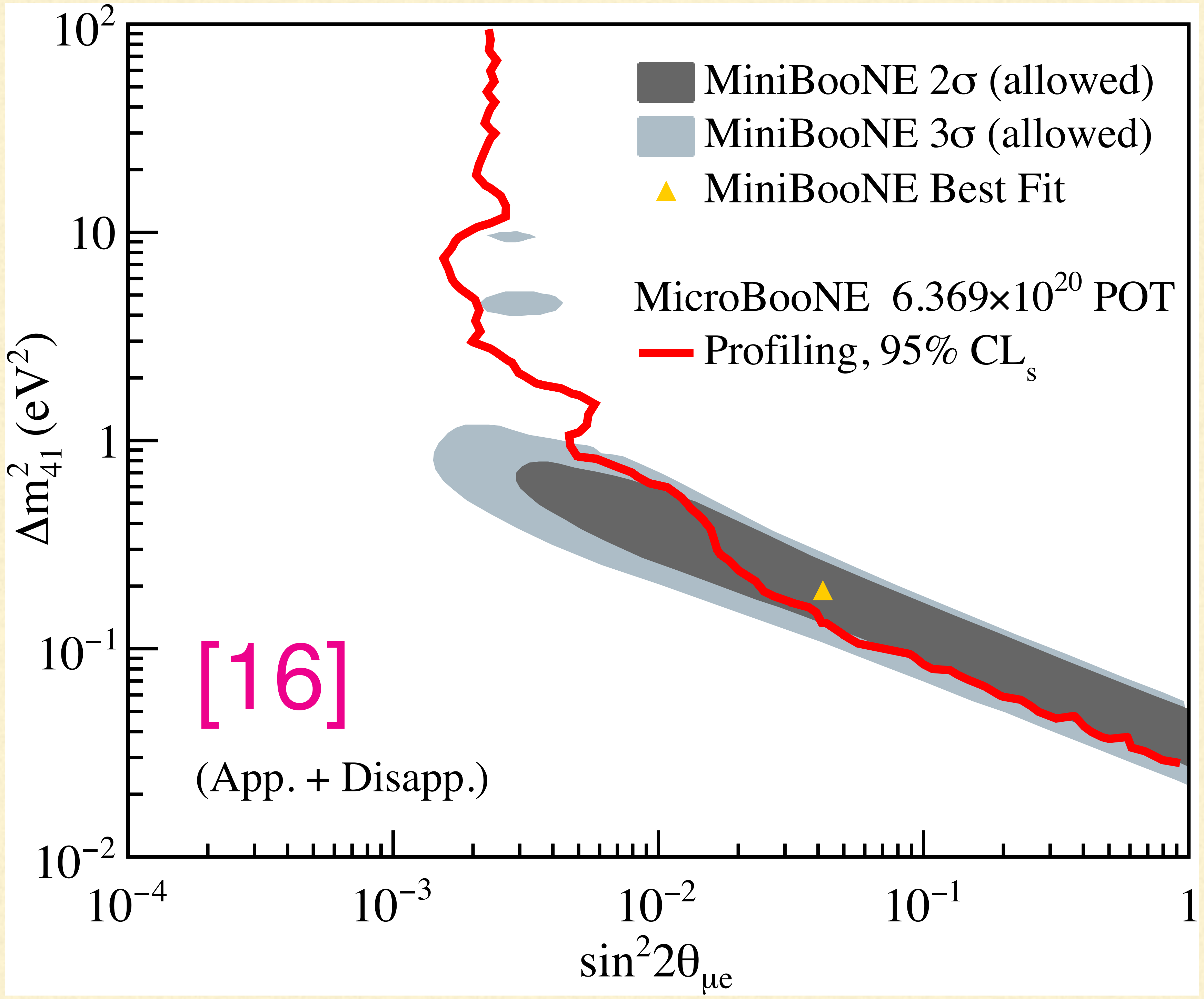


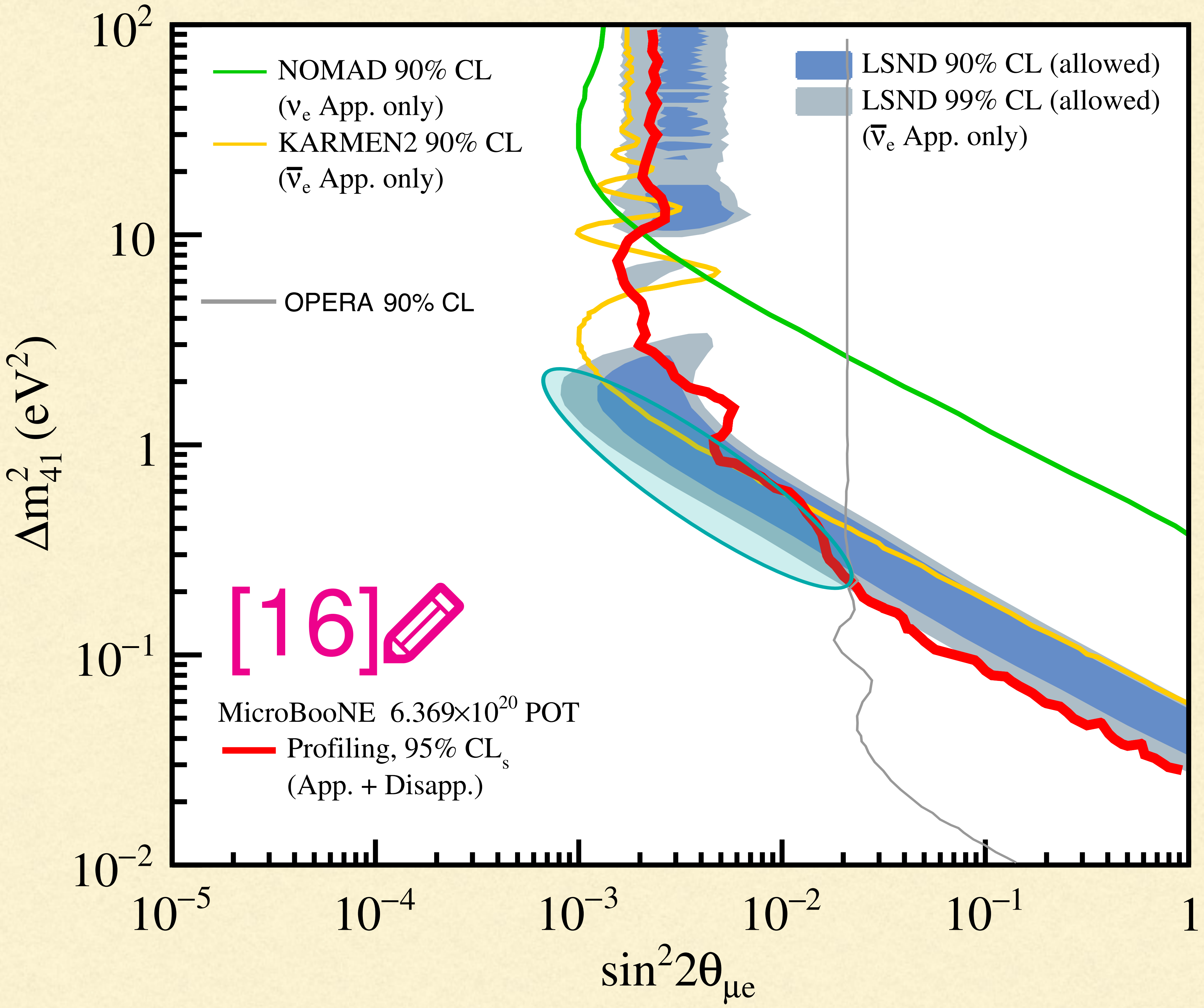
80 ton LAr TPC, L=468.5 m

Excellent particle identification capabilities.

Can potentially distinguish electrons , protons and photons

MicroBooNE ……….





MicroB has performed a 4ν analysis that disfavours much of MB/ LSND appearance space

Region now allowed ➡ $0.1 \lesssim \Delta m^2_{41}/\text{eV}^2 \lesssim 1$ and $10^{-3} \lesssim \sin^2\theta_{\mu e} \lesssim \text{few} \times 10^{-2}$

Maltoni, Nu 2024 talk

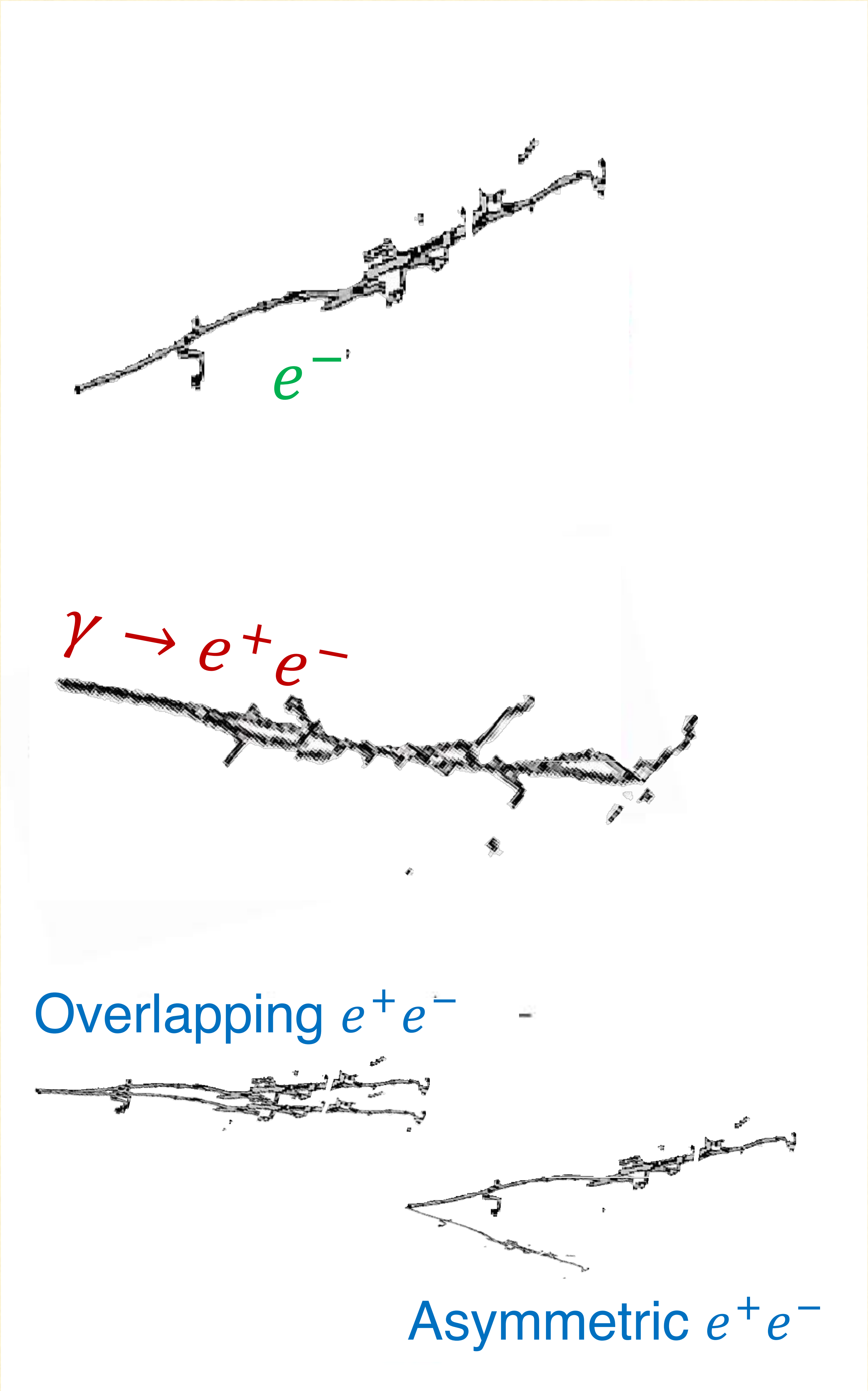
Some general comments…………

An important point: Both MB and LSND were mineral oil detectors measuring $E_{visible}$, unable to distinguish electrons from photons or e+e- pairs

New physics (NP) proposals rely on this limitation

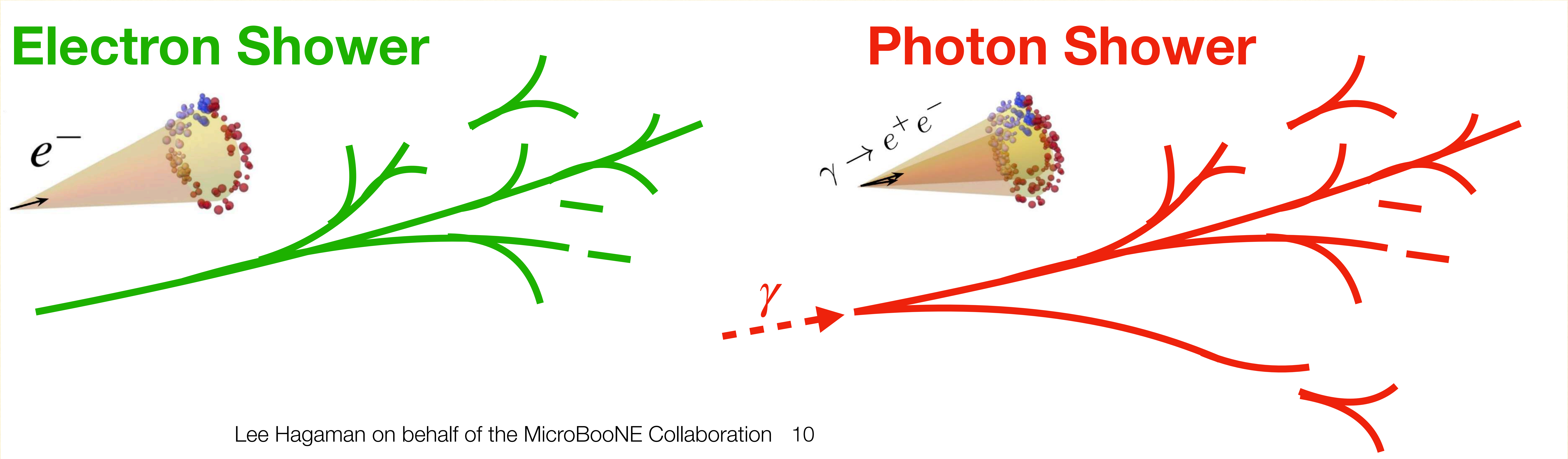
For a NP interaction giving an electron-like signal due to pair production in the LSND/MB detectors, a new mediator is required.

This can in principle be a vector, axial vector , scalar or pseudo scalar



How does MicroBooNE distinguish between an electron and a photon?

(And hence attempt to resolve the ambiguities inherent in the LSND/MiniBooNE results?)



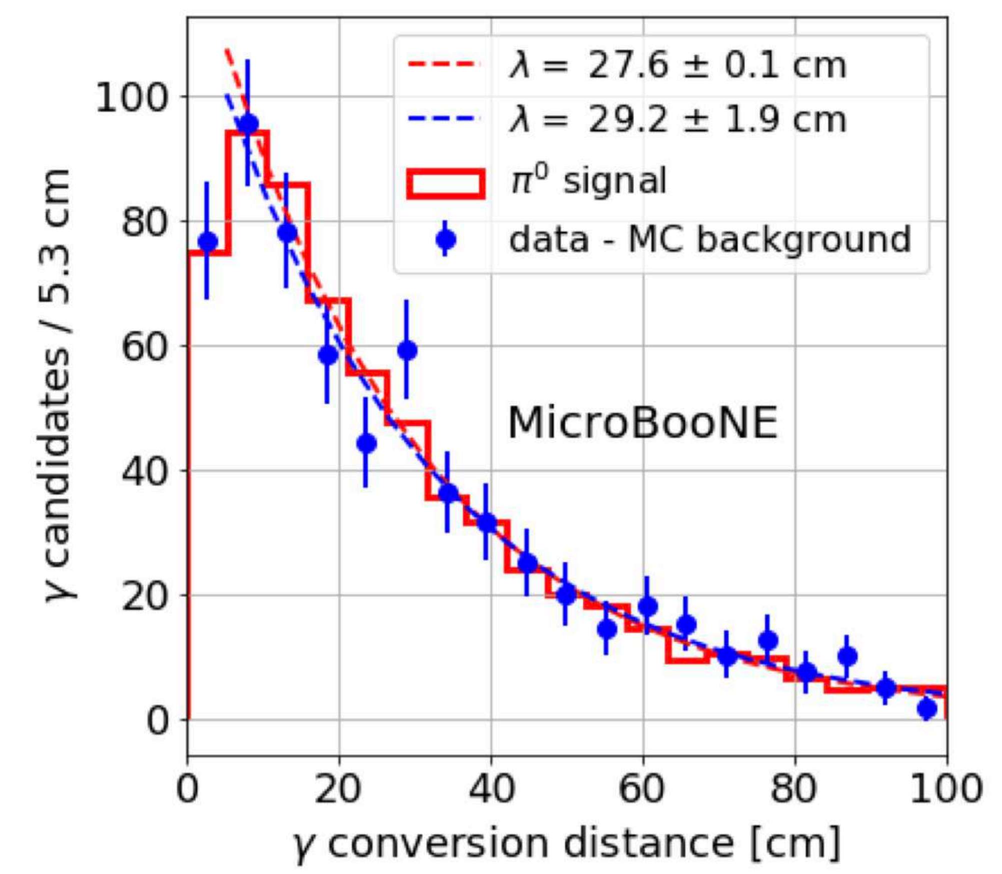
Electron Shower

Photon Shower

Lee Hagaman on behalf of the MicroBooNE Collaboration 10

How does MicroBooNE distinguish between an electron and a photon?

- They look for gaps between the electromagnetic shower and hadronic activity
  - Photons will tend to have a gap of ~25 cm before pair converting and showering



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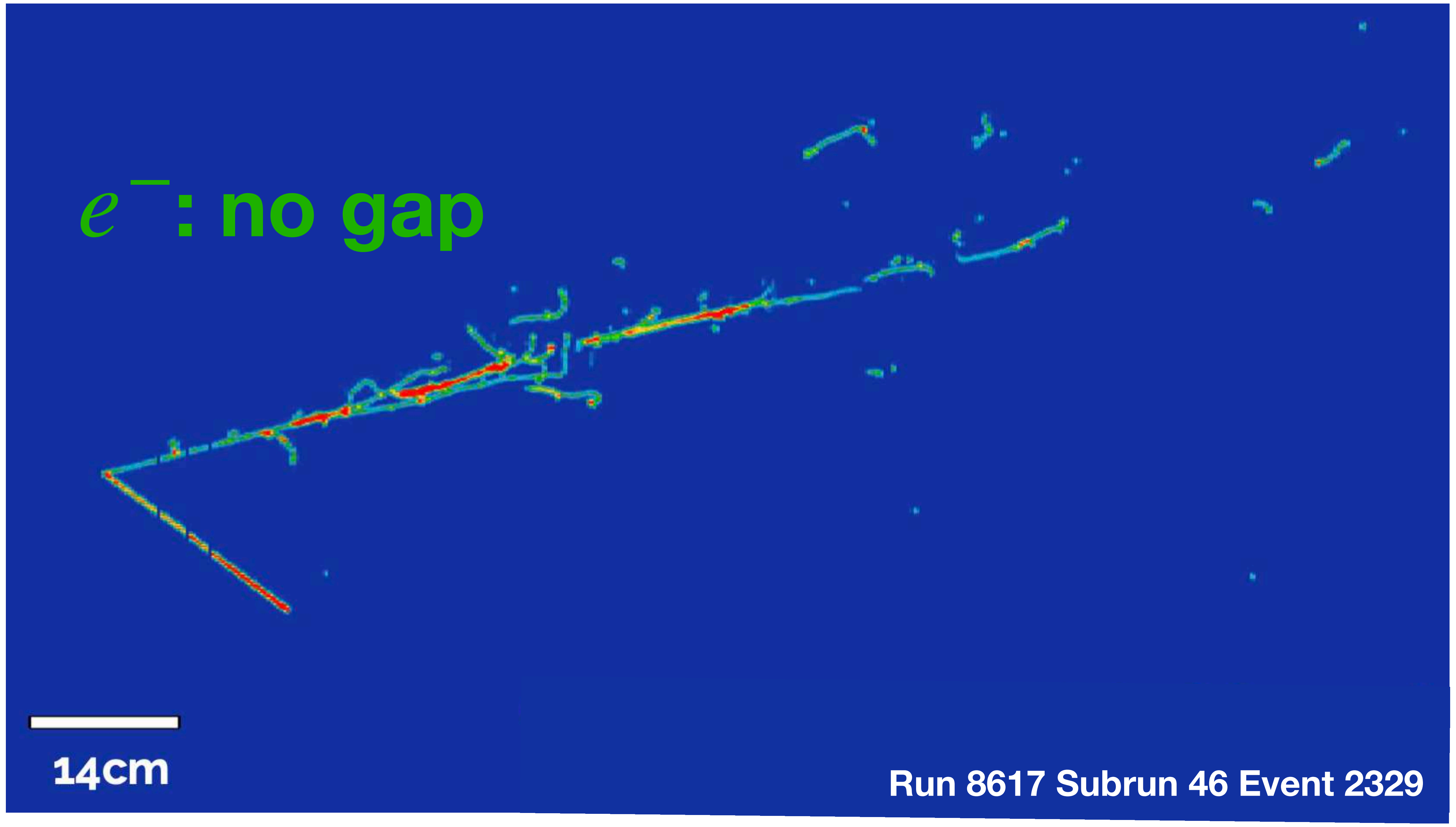
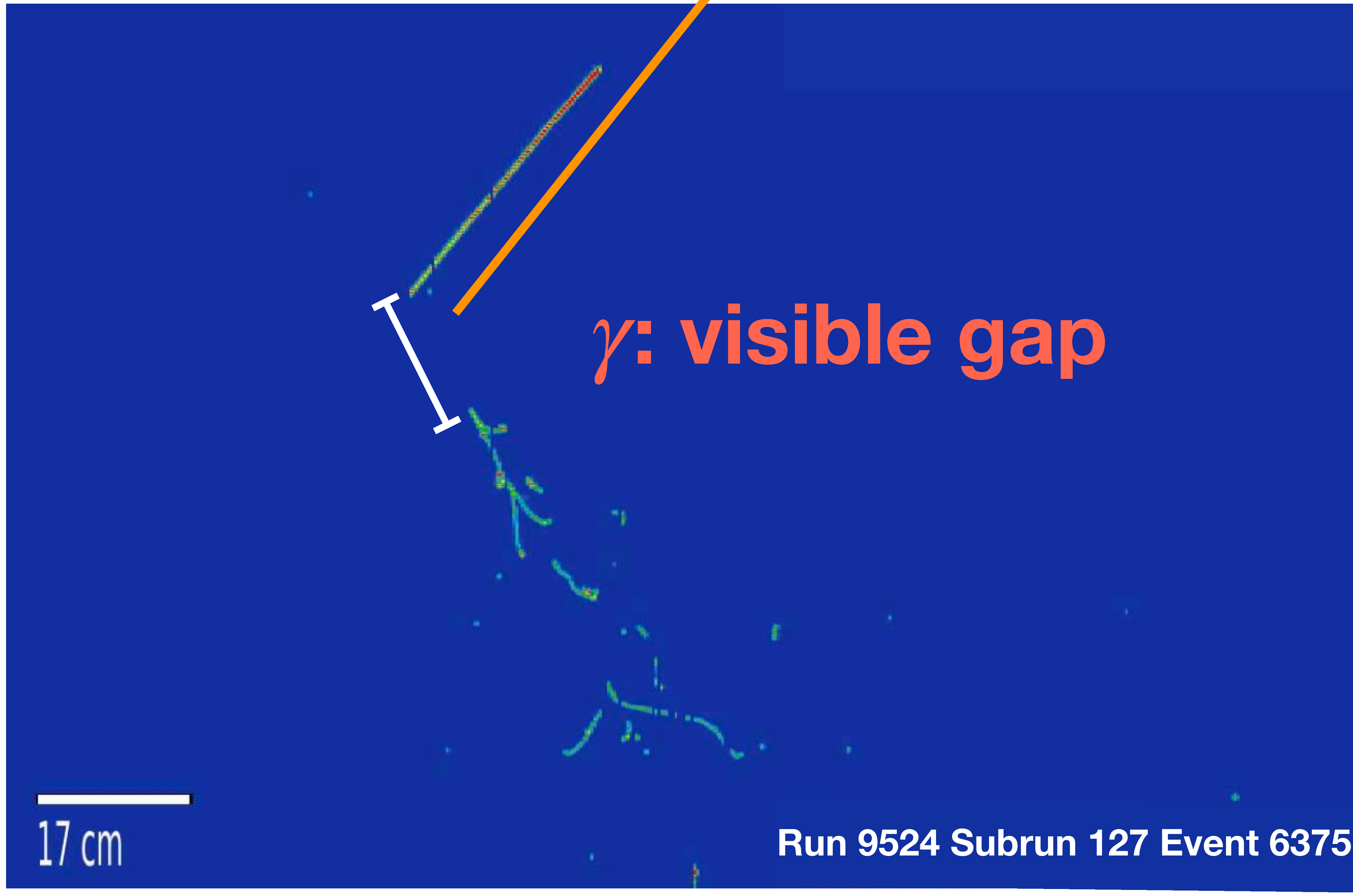
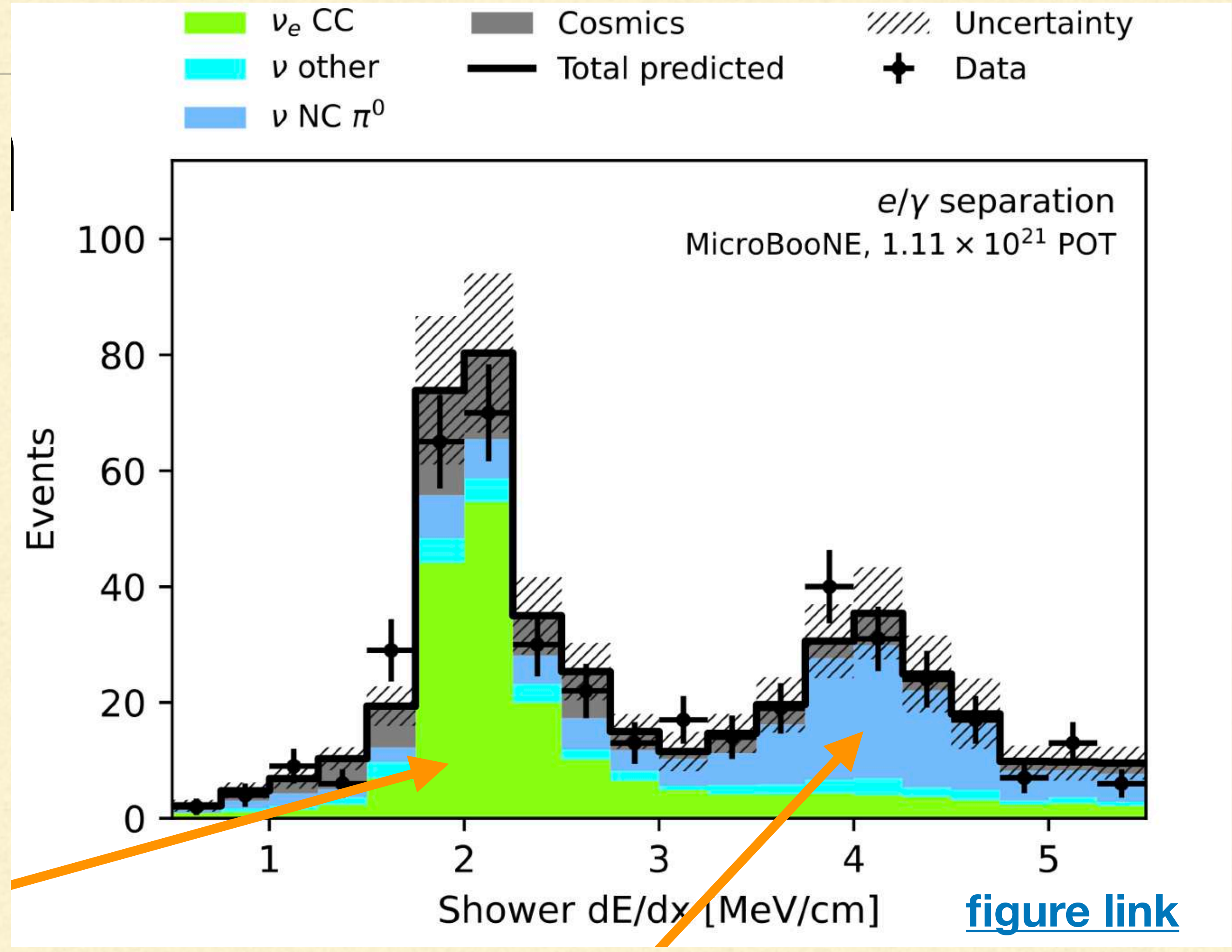

e^-: no gap

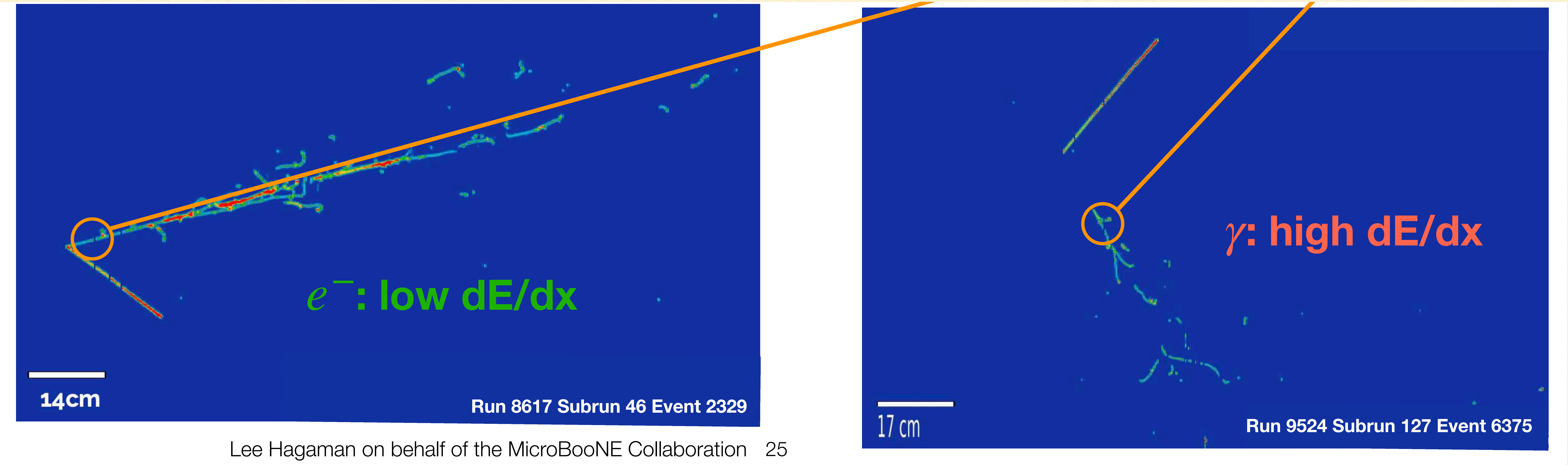

γ: visible gap

How does MicroBooNE distinguish between an electron and a photon?

- The deposited energy per unit length in the shower stem also lets them distinguish electrons from photons

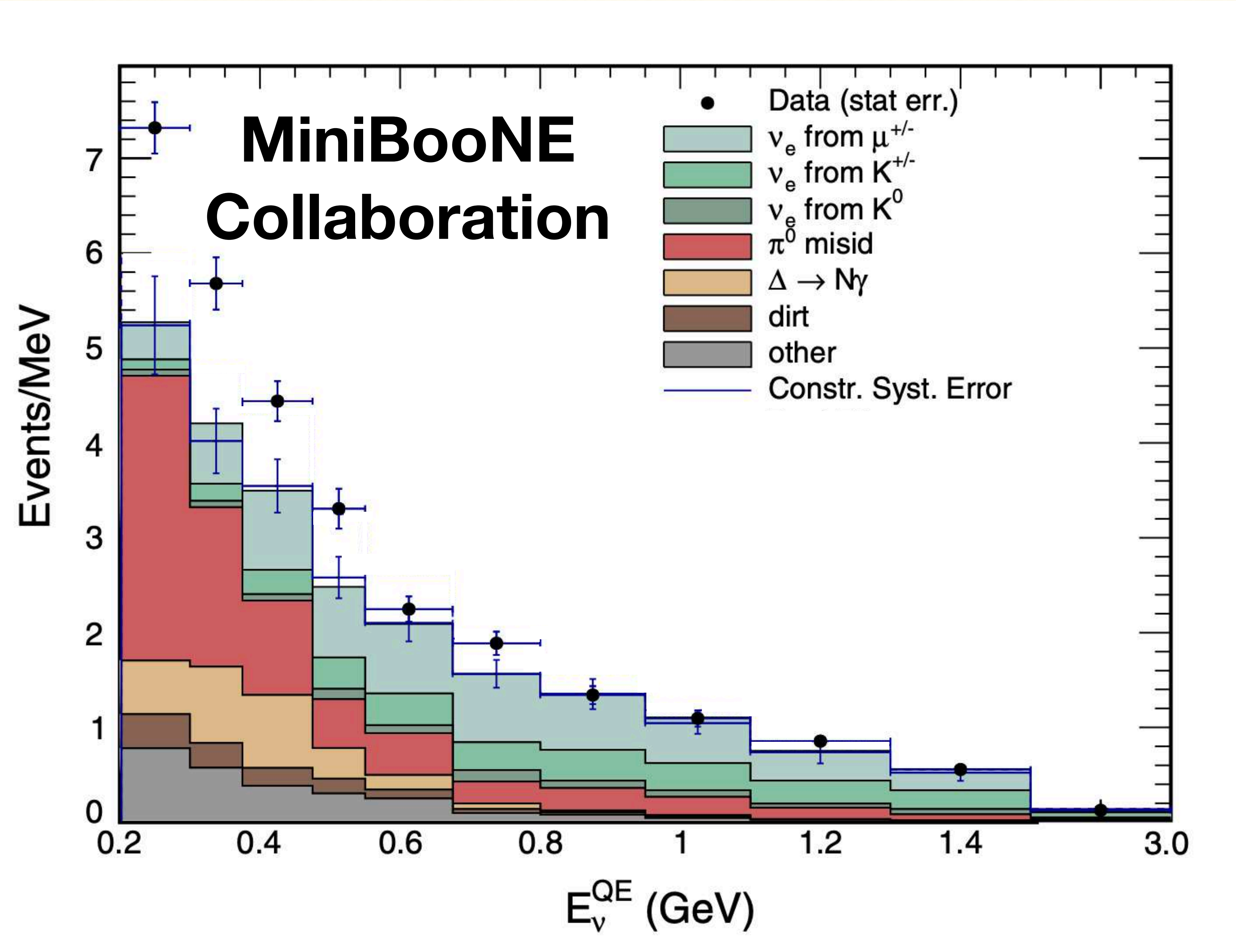
e+e- deposits twice as much energy as e-

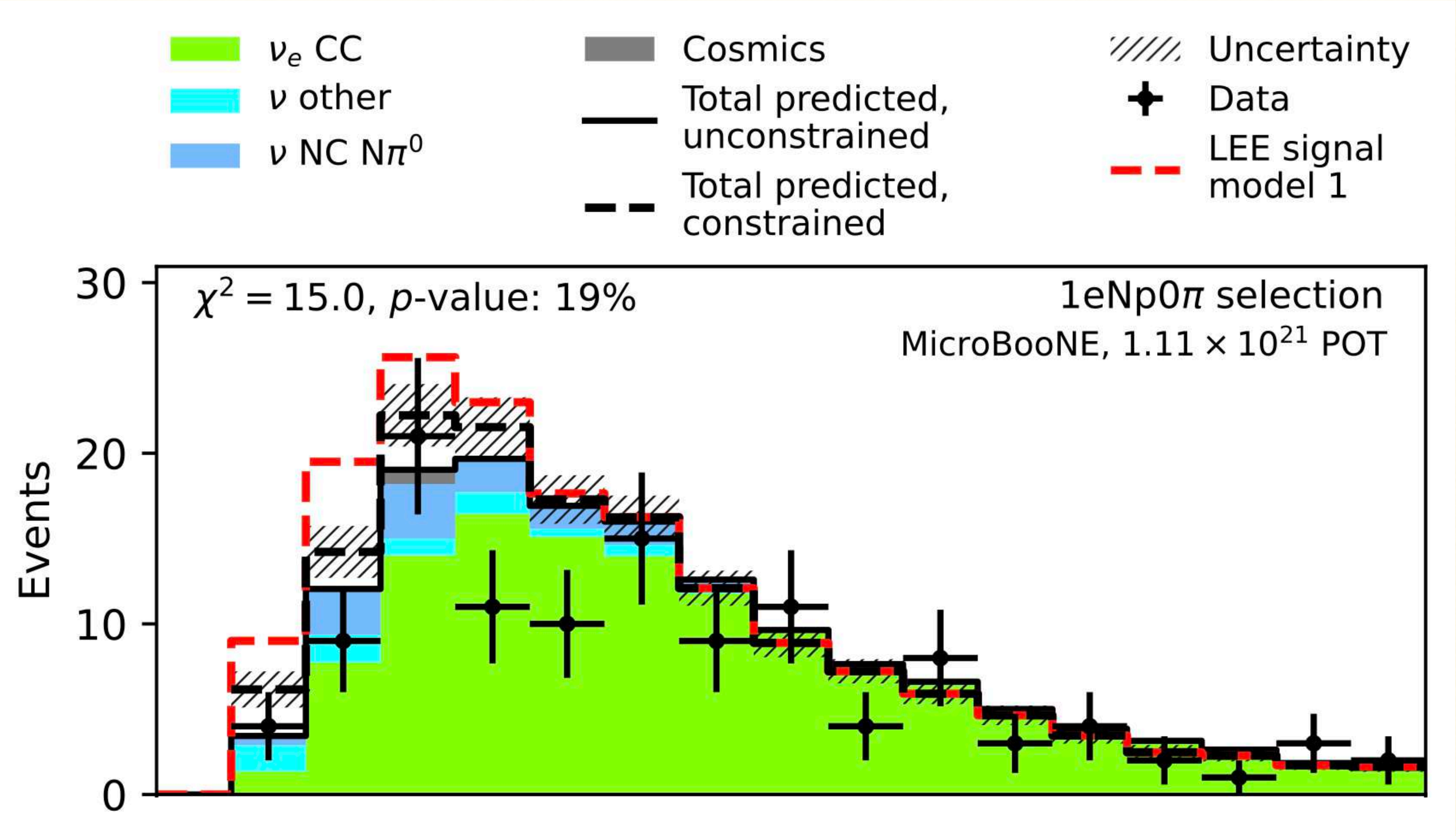




Lee Hagaman on behalf of the MicroBooNE Collaboration 25

Checking if the MiniBooNE excess was due to electrons …….



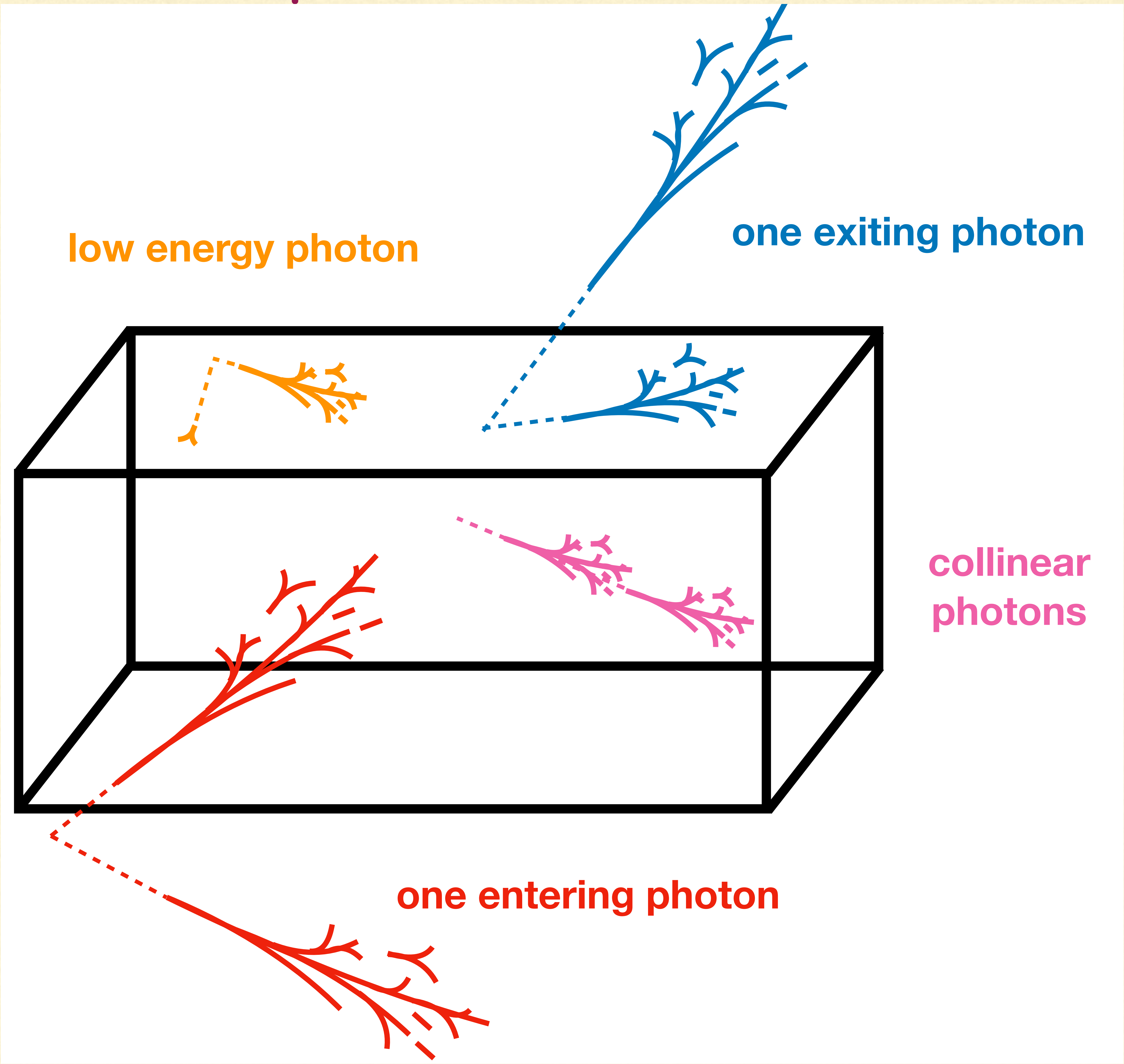


MicroBooNE Collab, arXiv:2412.14407)

MicroBooNE has found no evidence for any electron excess. This constrains the active-sterile oscillation scenario and other NP interactions which lead to electron production.

Checking if the MiniBooNE excess was due to photons …….

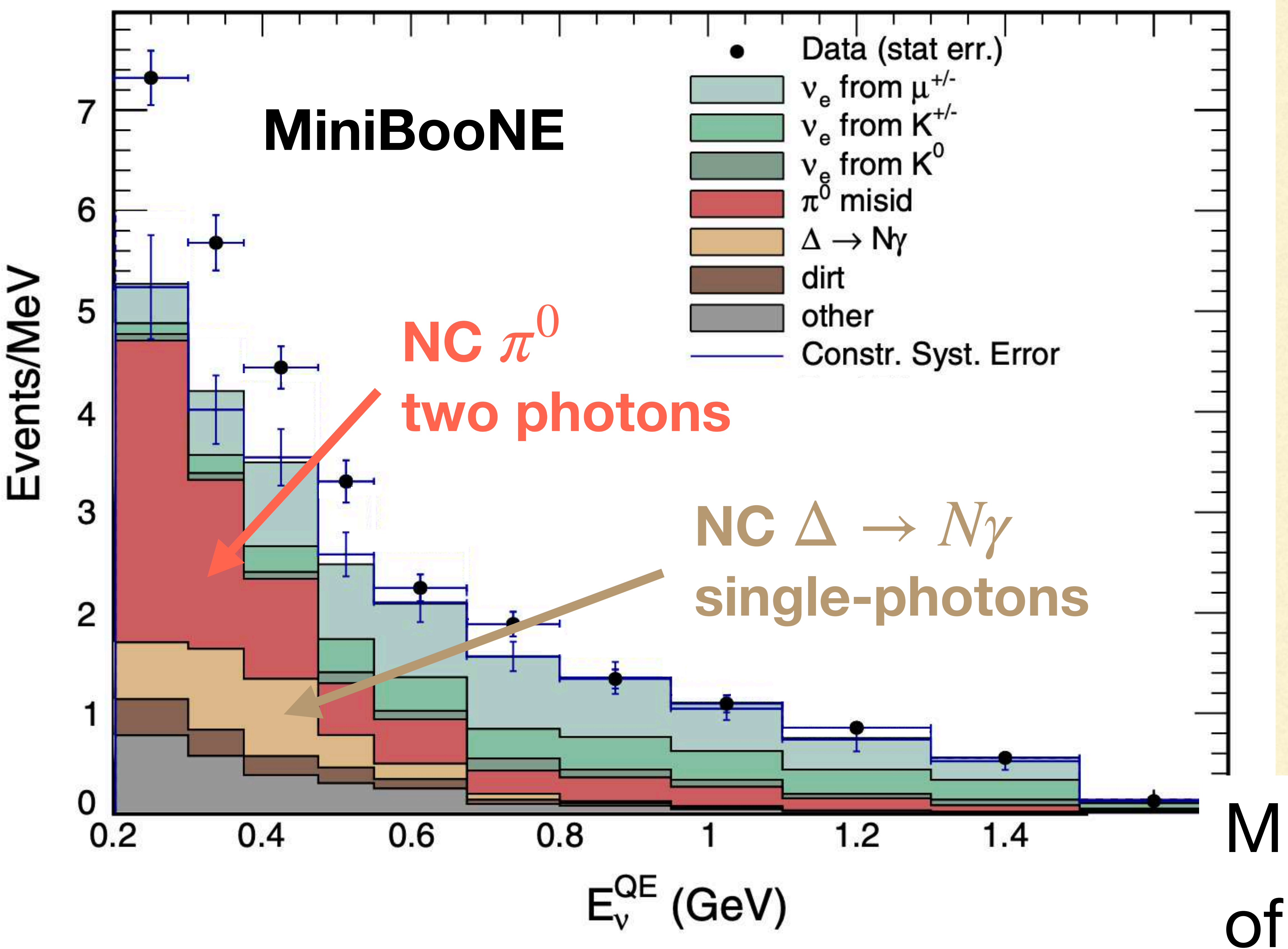
This determination is harder than that for electrons, mainly due to $\pi^0 \rightarrow \gamma + \gamma$ backgrounds.



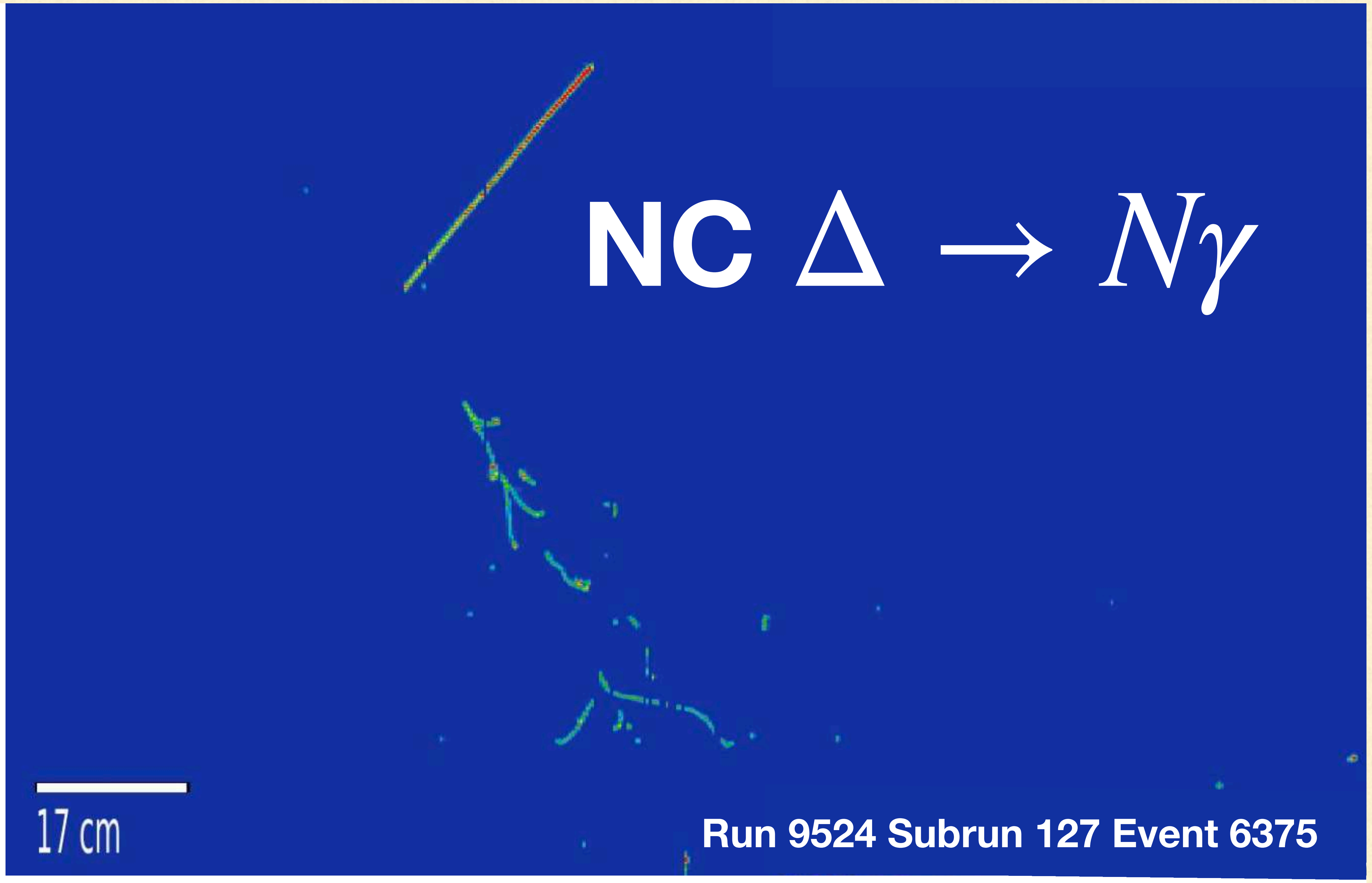
How may genuine signal single photons be produced leading to the MB excess? …….

Phys. Rev. Lett. 128, 111801 (2022)

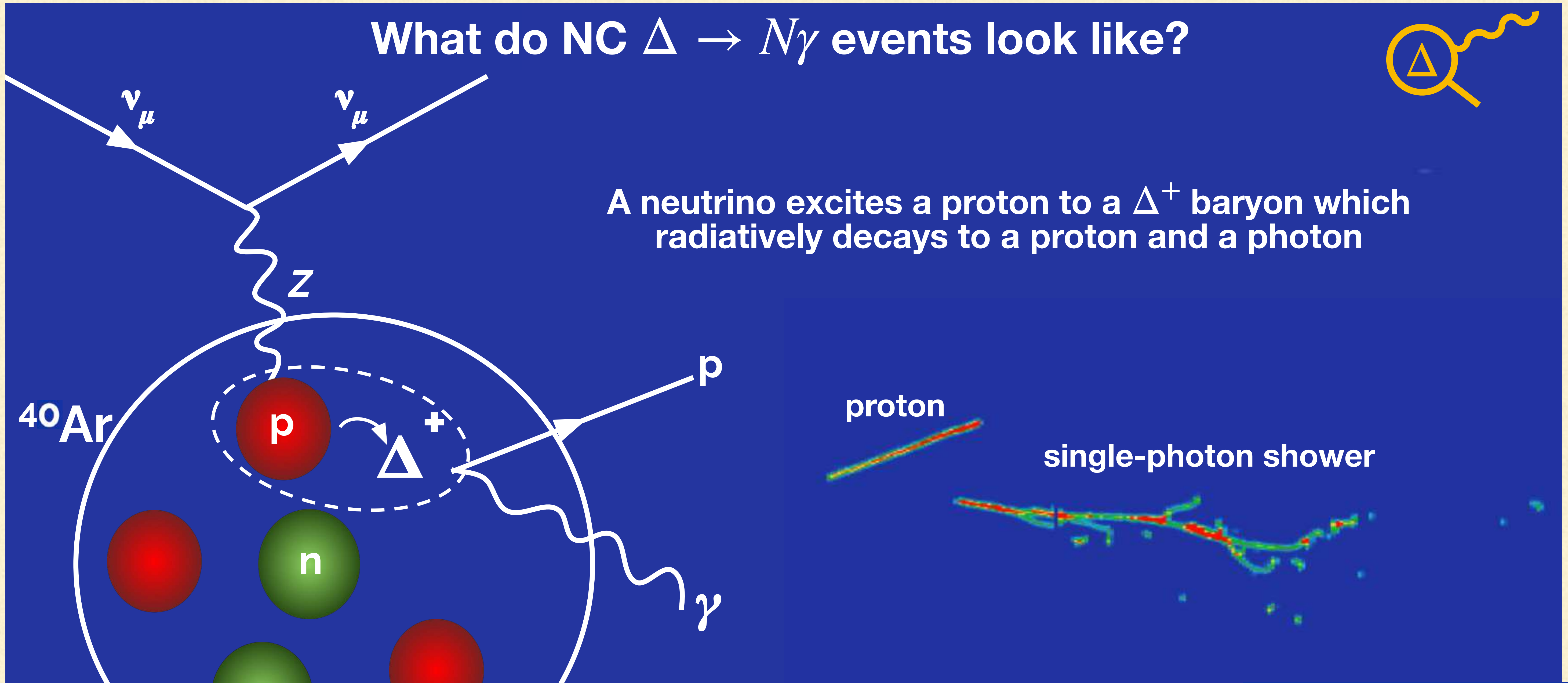
NC $\Delta \rightarrow N + \gamma$ Single-Photons



Phys. Rev. D 103, 052002 (2021)

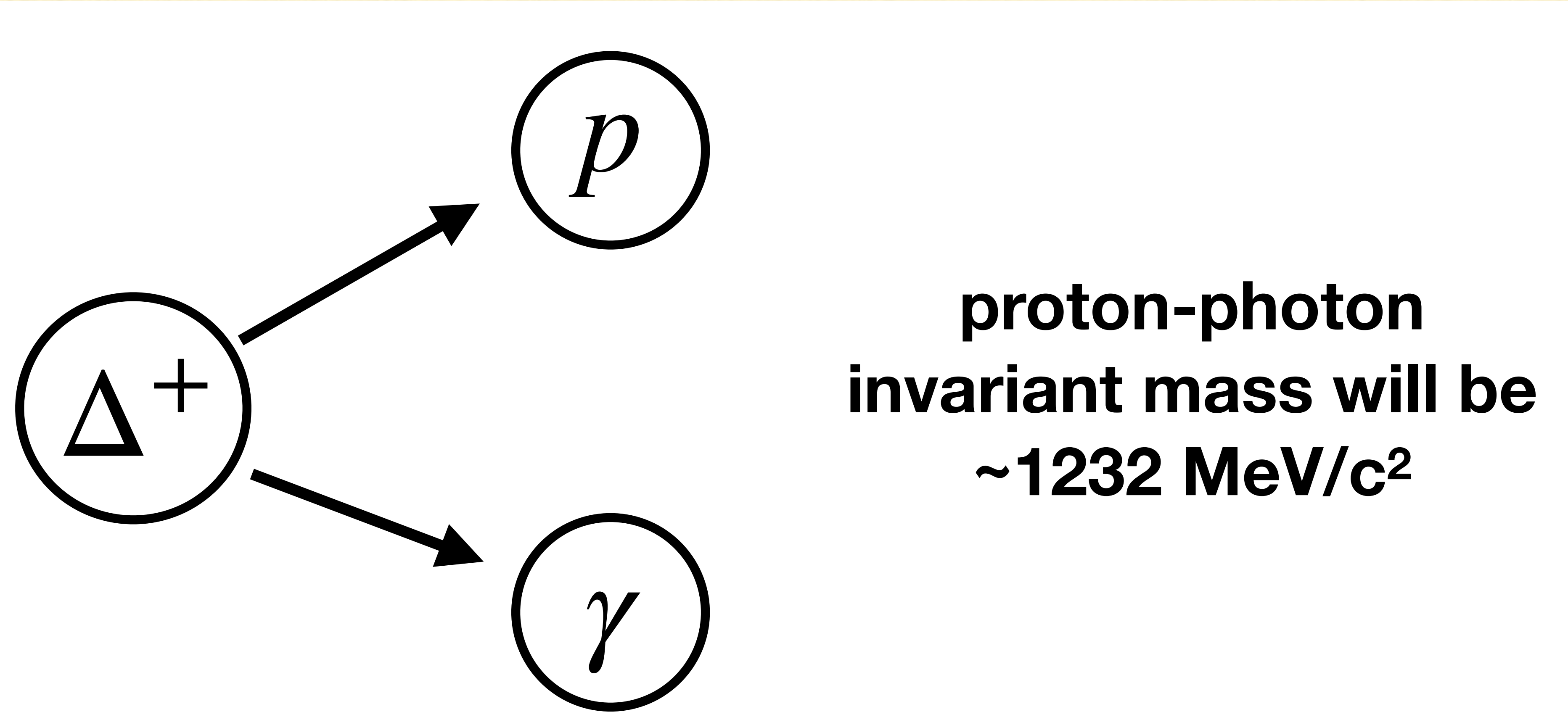


MiniBooNE reported that a 3.18x enhanced rate of NC $\Delta \rightarrow N\gamma$ events would explain the excess well

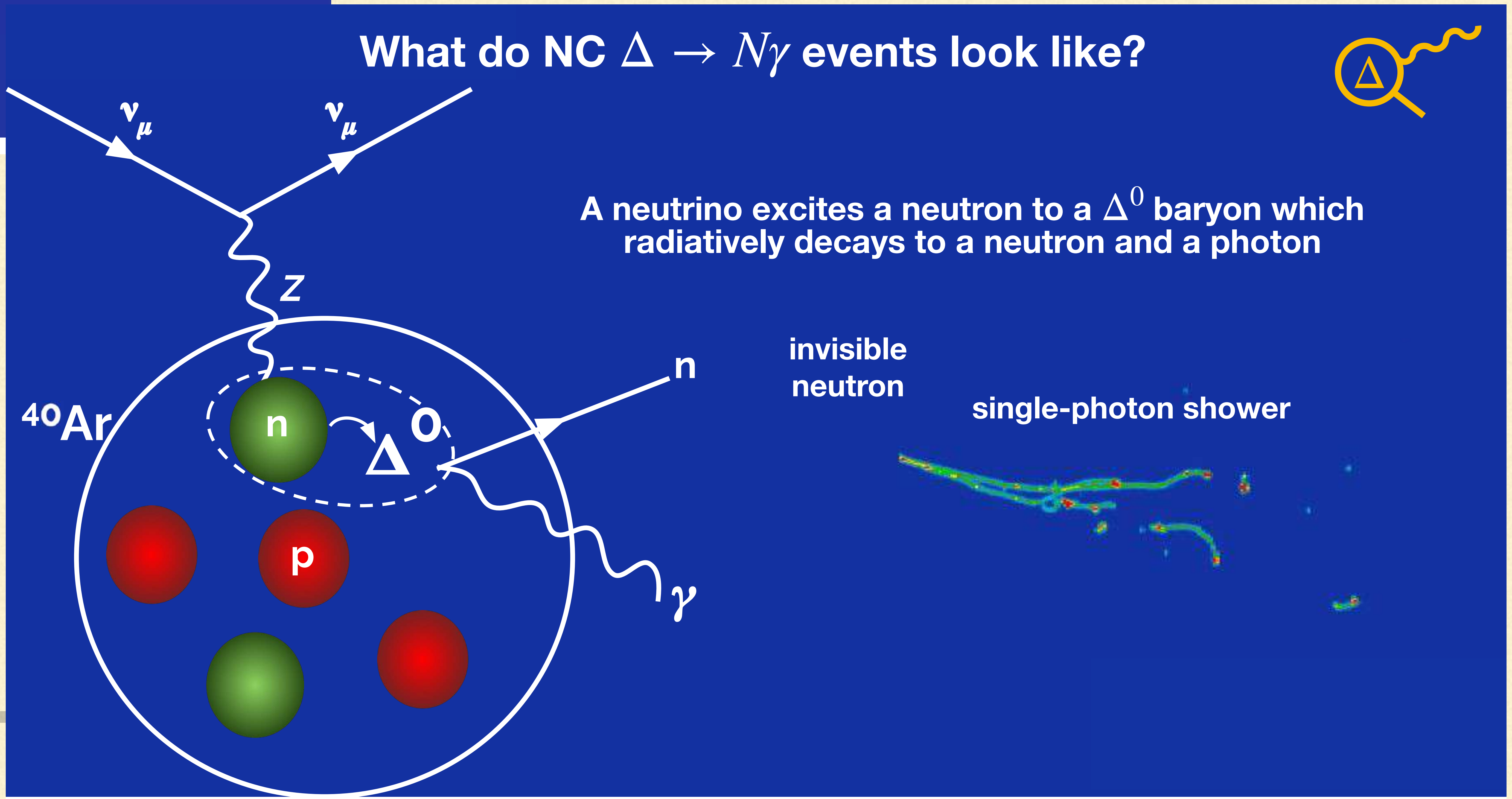


What do NC $\Delta \to N\gamma$ events look like?

A neutrino excites a proton to a Δ^+ baryon which radiatively decays to a proton and a photon



proton-photon invariant mass will be ~1232 MeV/c²

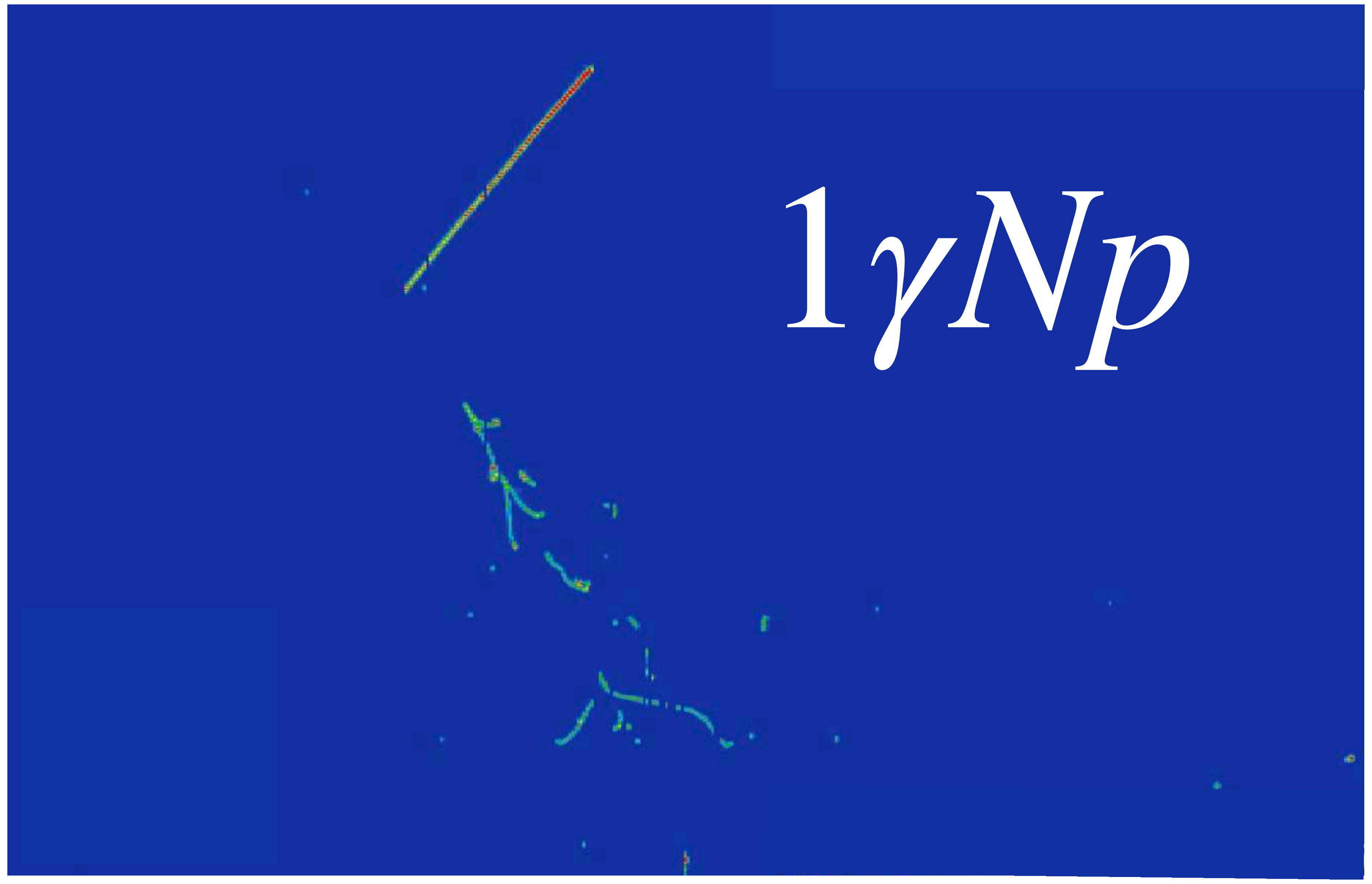


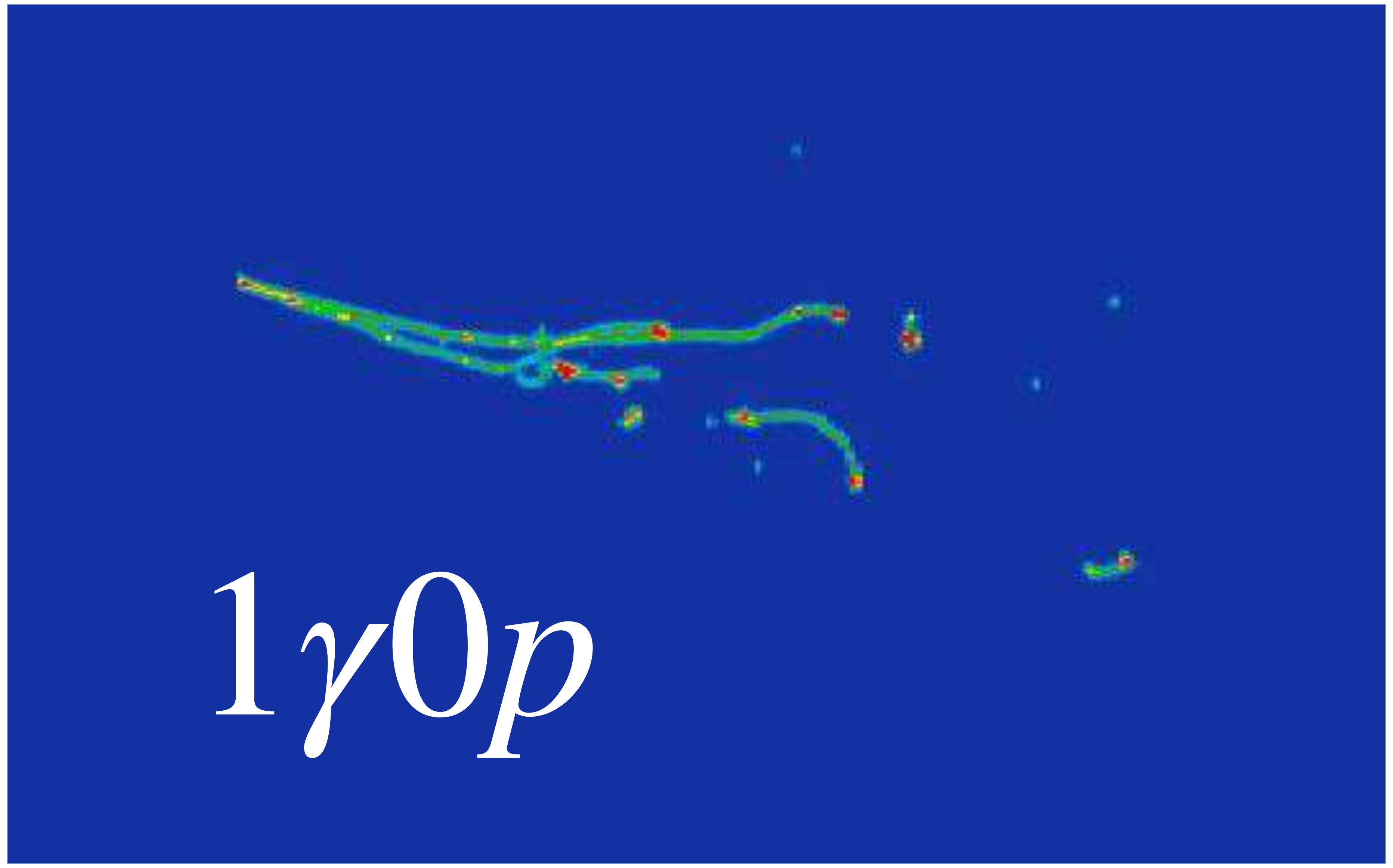
What do NC $\Delta \to N\gamma$ events look like?

A neutrino excites a neutron to a Δ^0 baryon which radiatively decays to a neutron and a photon

The importance of looking at both 0p and 1p processes …….

In the various photon searches, MicroBooNE focussed on both 1gamma1p as well as 1gamma0p, because MiniBooNE was not capable of seeing protons, hence a gamma based excess could come from either of these processes.



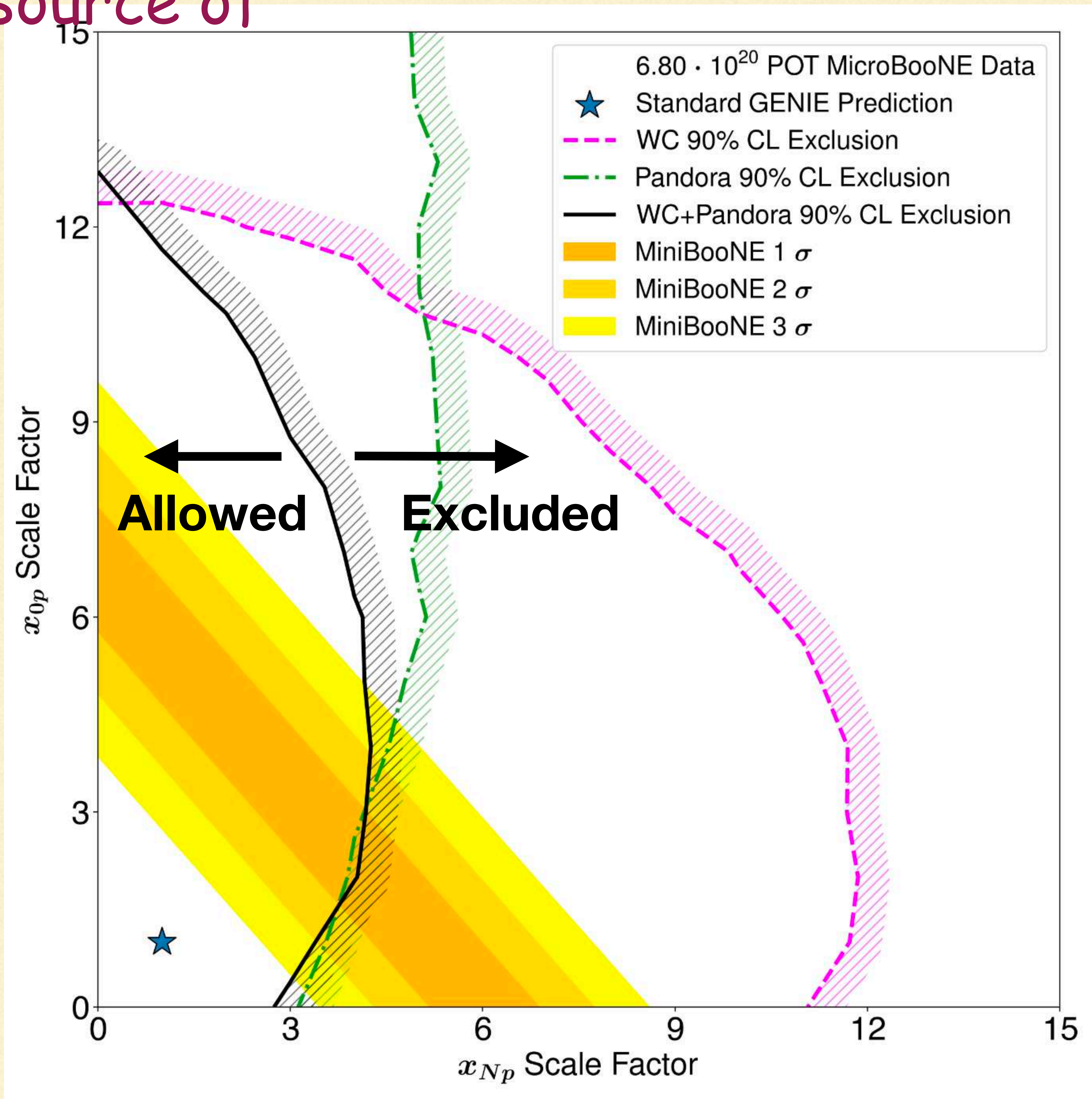


MicroBooNE results so far on photons as a source of MiniBooNE LEE …….

MicroBooNE excludes the hypothesis that the MiniBooNE LEE is explained by A factor 3.18 enhancement of the $\text{NC } \Delta \rightarrow N\gamma$ rate at 94.4% CL

MicroBooNE excludes the hypotheis that the MiniBooNE LEE consists of mostly 1γNp events.

- MicroBooNE, however, does not exclude a MiniBooNE LEE hypothesis consisting of mostly 1γ0p



Scale factors x_{0p}, x_{NP} represent the multipliers over the GENIE (SM) prediction.

Additional MicroBooNE results on photons as a source of MiniBooNE LEE…NC Coherent single photons

10x rarer than NC $\Delta \rightarrow N\gamma$,

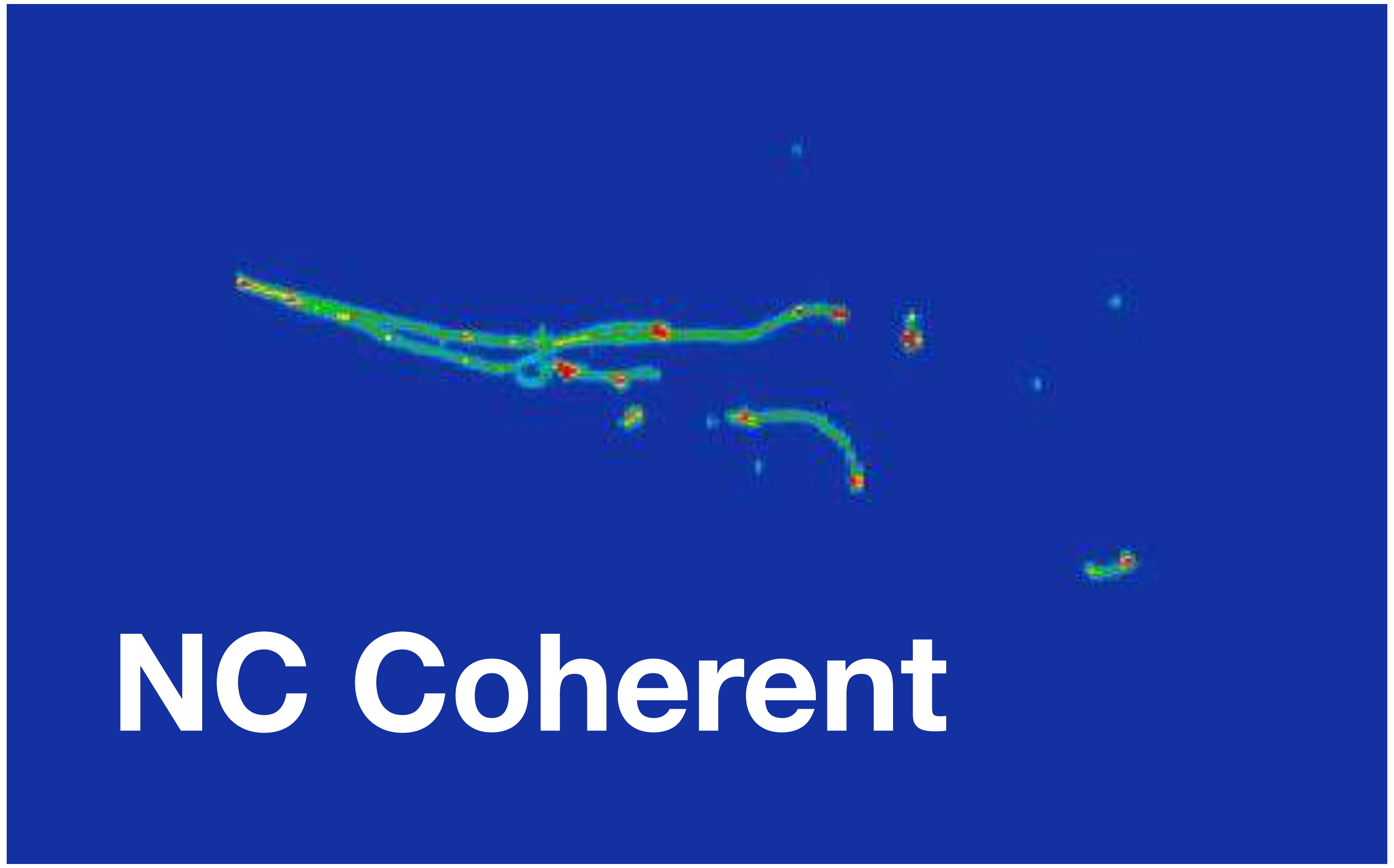
Very rare process in SM, but if there is an anomalously large rate due to NP, MicroBooNE should see it.

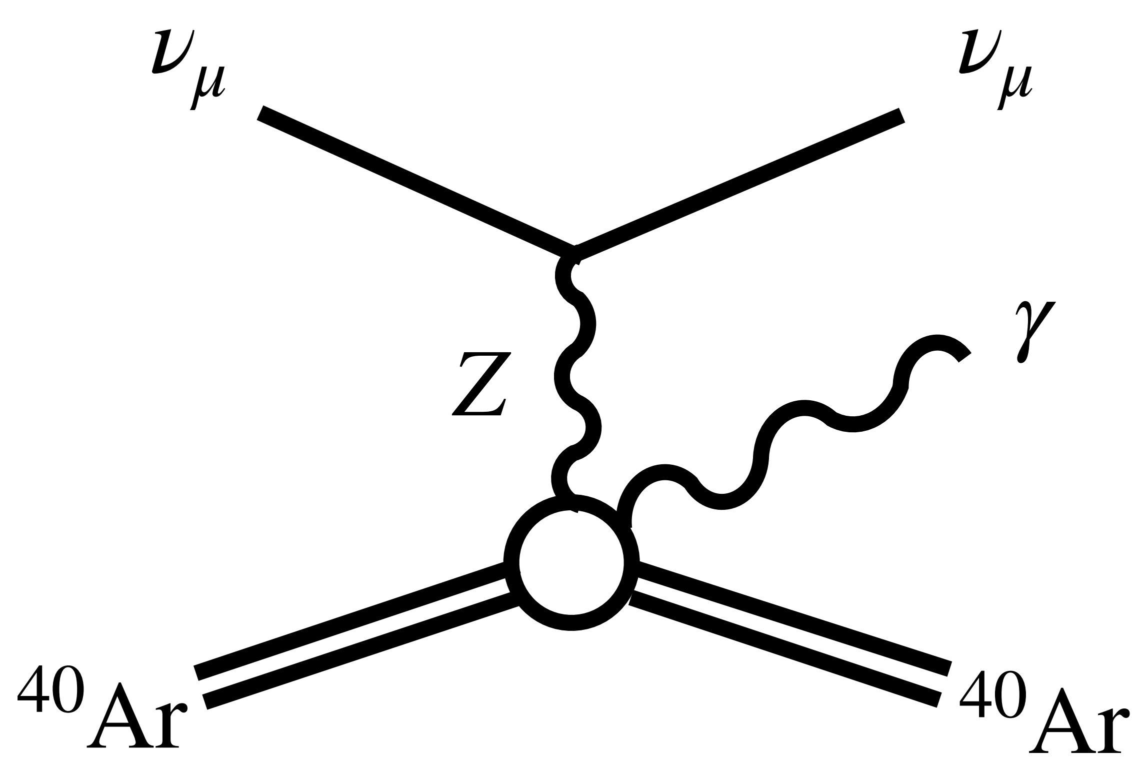
What is it?

The neutrino interacts with the entire nucleus at once, with no detected hadronic activity. The nucleus remains in its ground state after the interaction, with a single emitted photon.

Due to this, the photon is expected to be fairly forward, and carry away low energy.

This first search focussed on a zero proton topology and found no excess

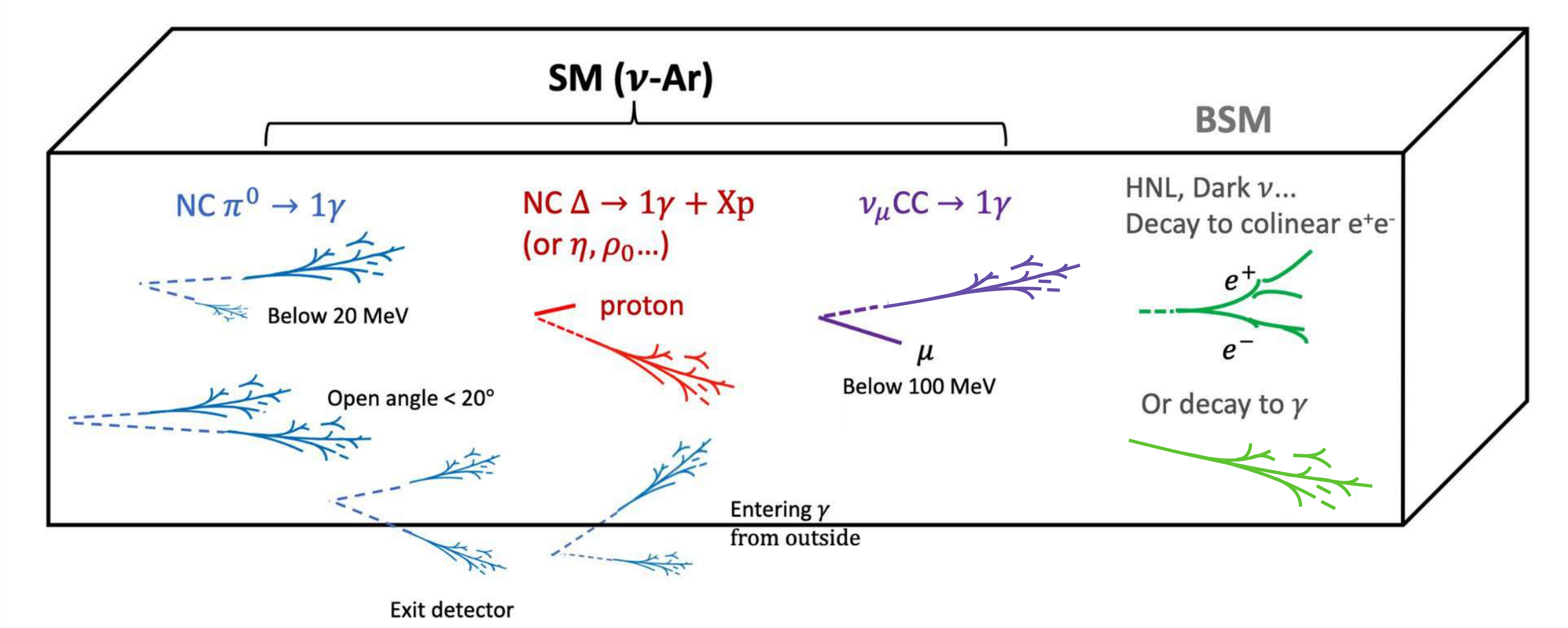




MicroBooNE Collaboration, 2502.06091

Additional MicroBooNE results on photons as a source of MiniBooNE LEE……………Inclusive single photon search

Inclusive single-photon search is broad-based in order to capture any potential single-photon anomaly

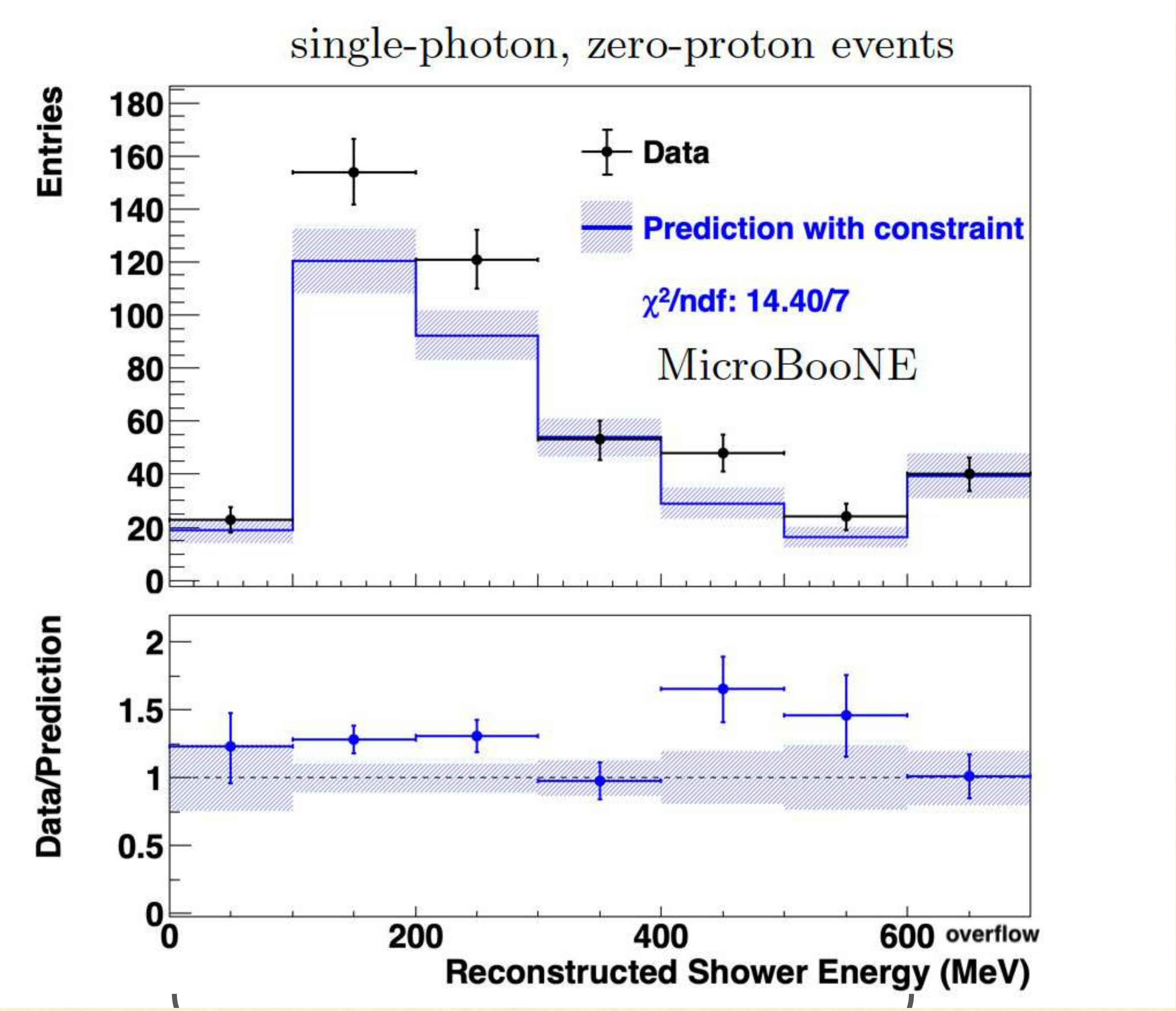


Lee Hagaman on behalf of the MicroBooNE Collaboration, Fermilab W&C seminar Feb 8, 2025

Additional MicroBooNE results on photons as a source of MiniBooNE LEE……………Inclusive single photon search

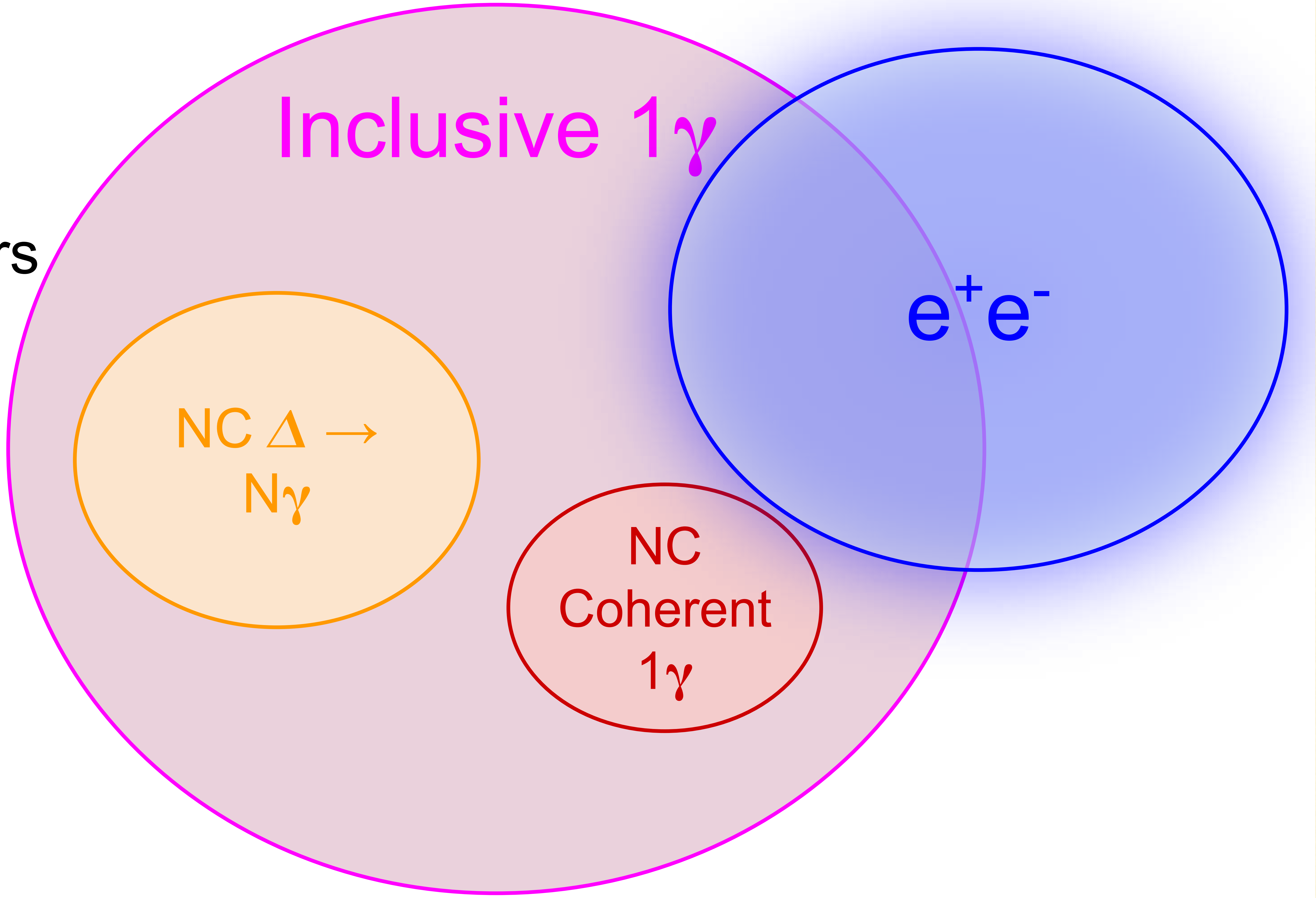
Full Search shows agreement at 1.6 sigma

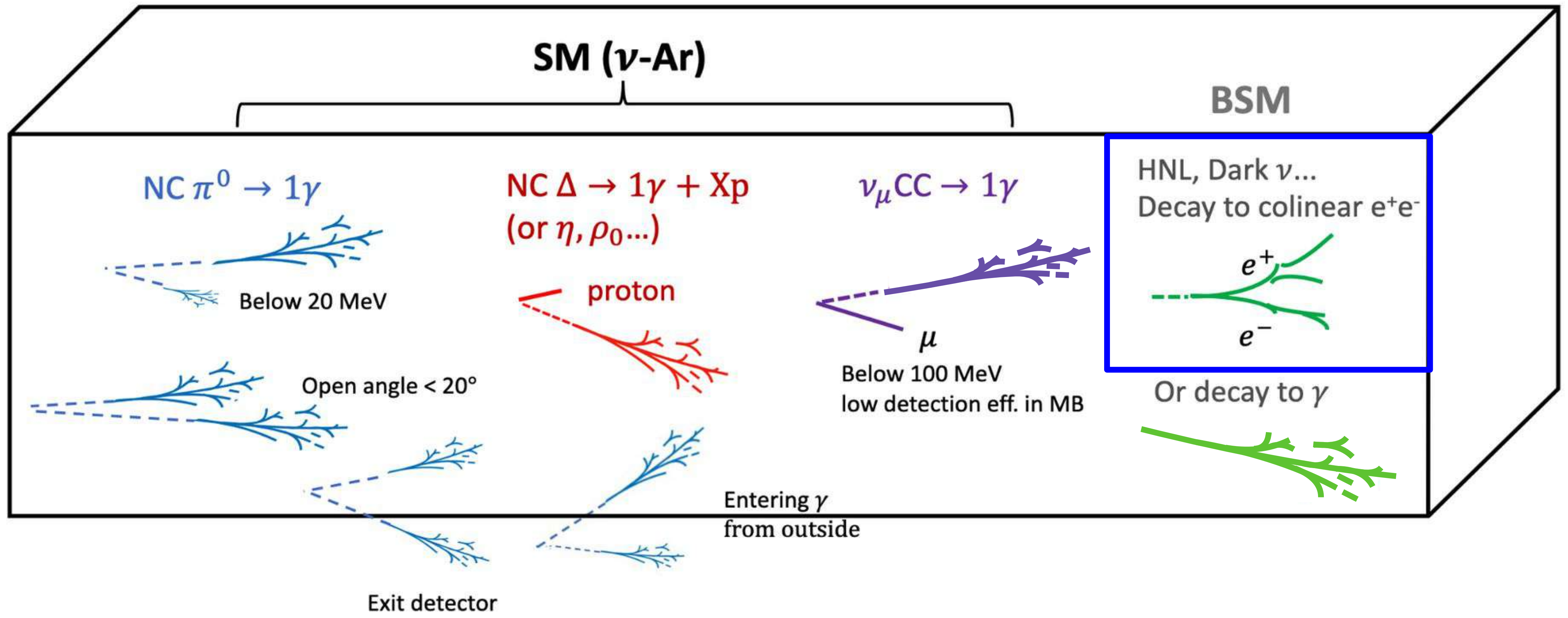
In the sub-sample with zero protons, there is a 2 sigma excess in events below 600 MeV

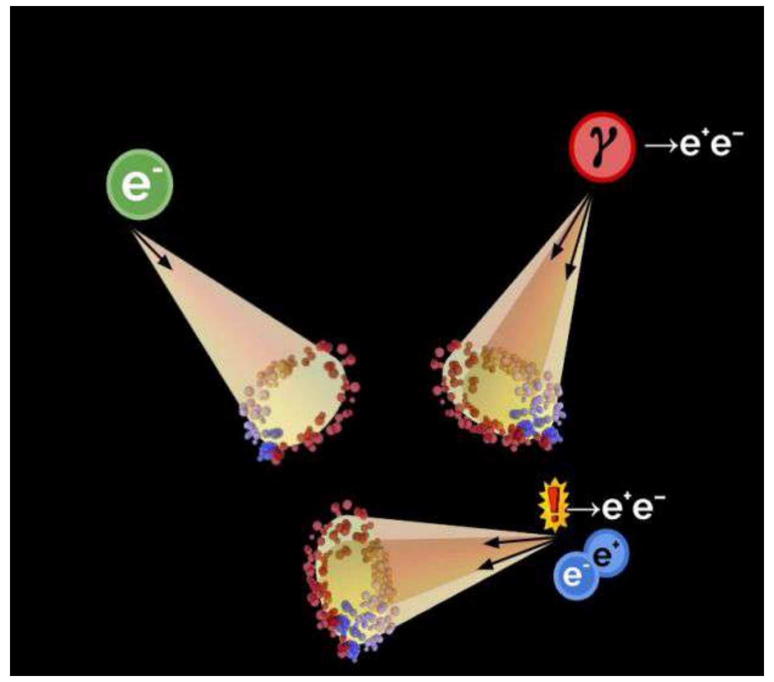


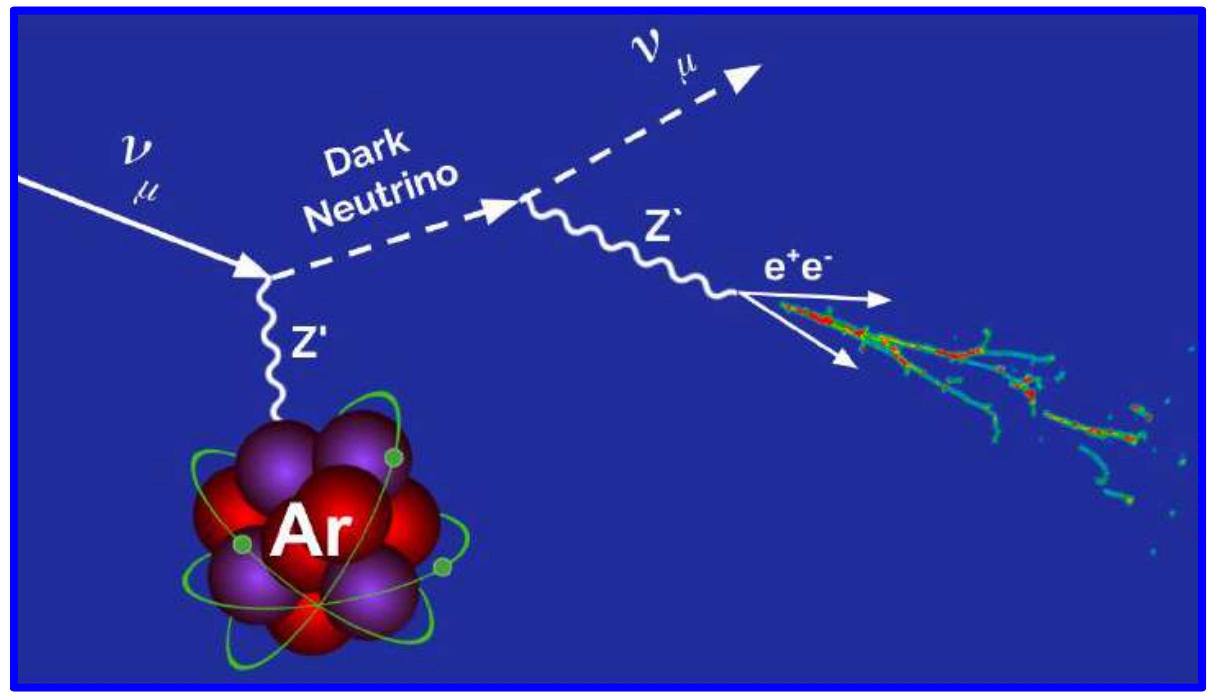
What Next?

- Look into a different topology that can often *look* like a photon: electron-positron pairs







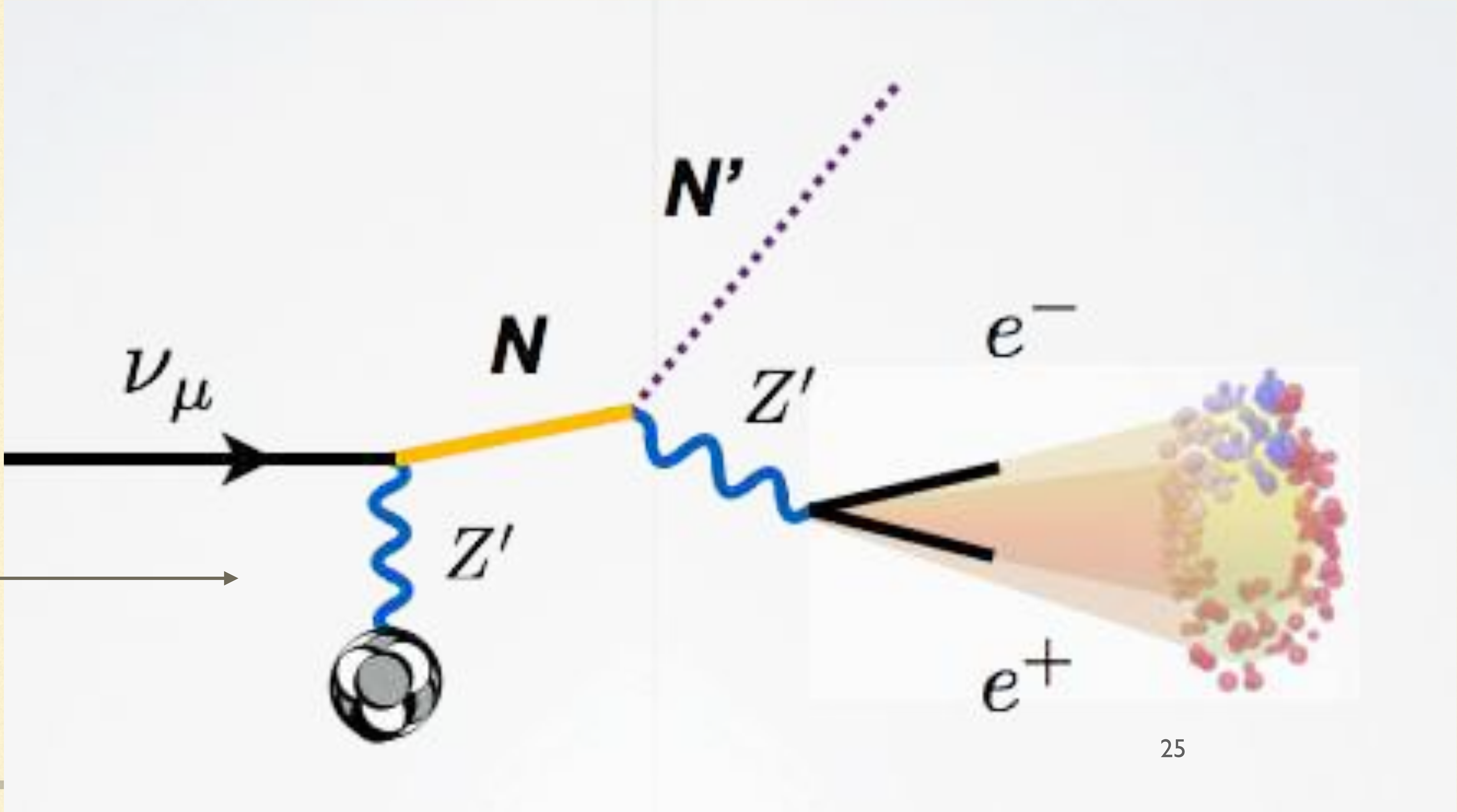


New Physics solutions to MB which lead to e+e- production

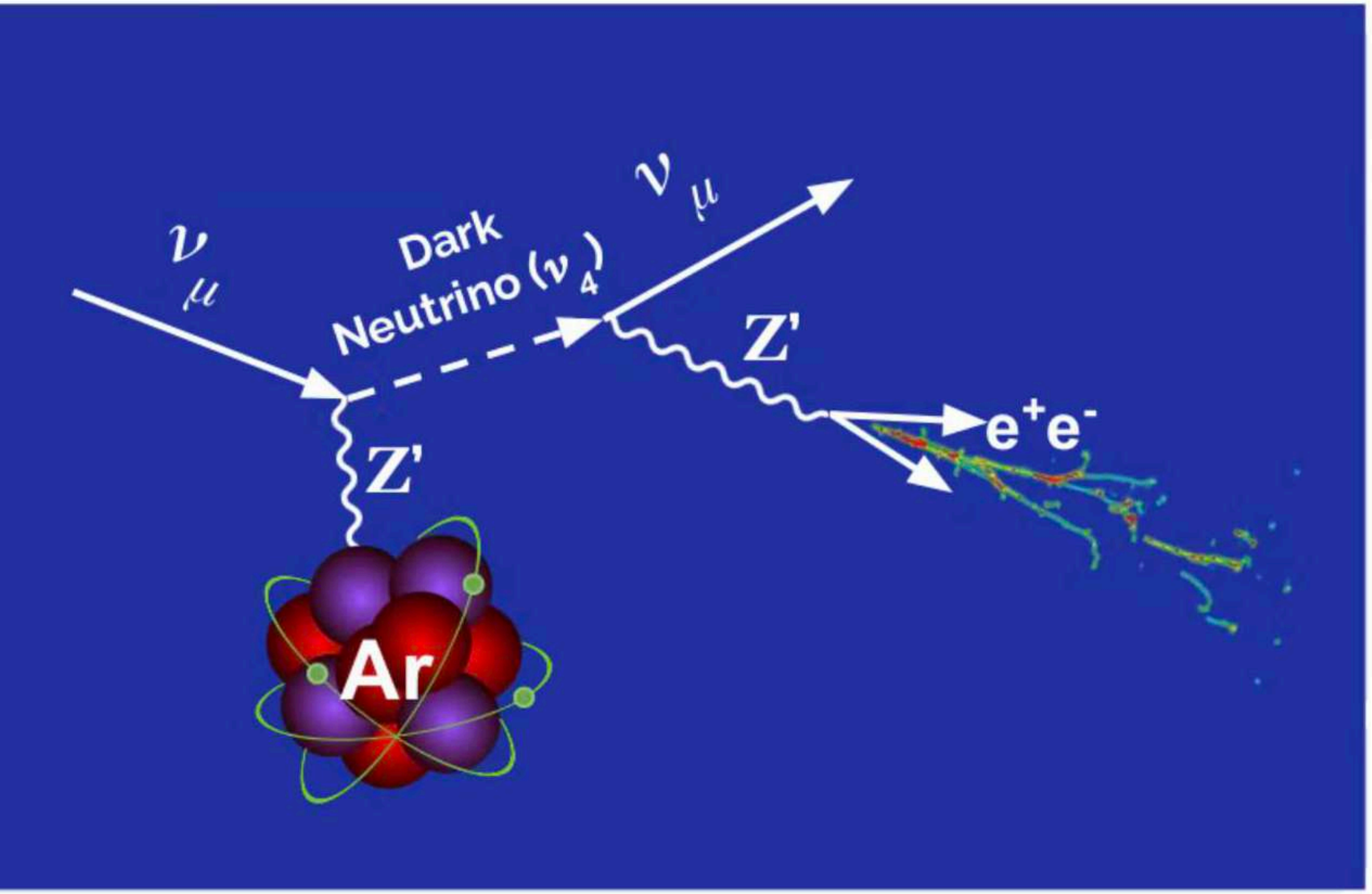
- Generic new physics process
- Mediator in general can be a vector, axial vector, scalar or pseudo scalar

NSI, but at low energies

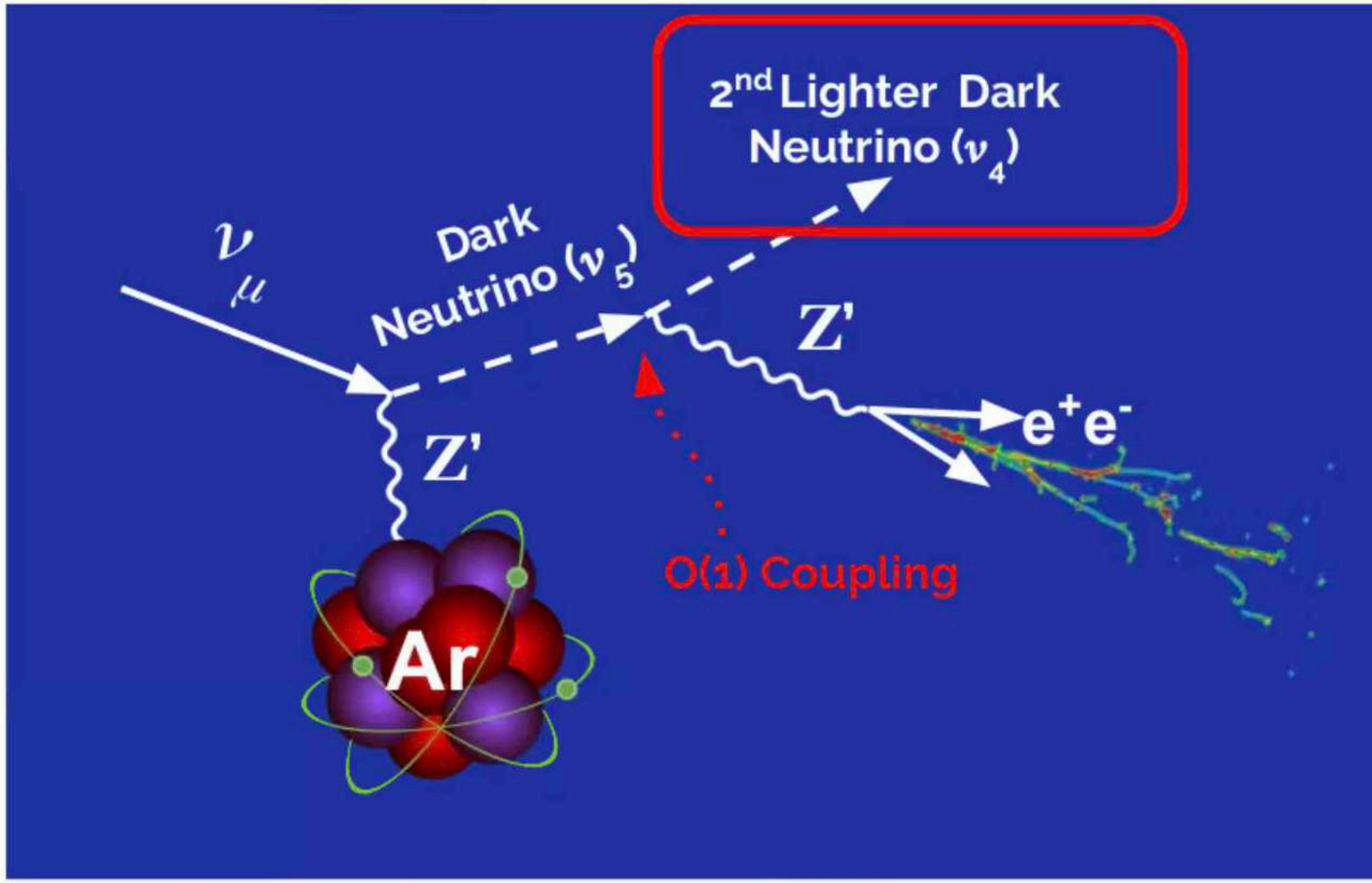
Z', both heavy and light, is the focus of latest search



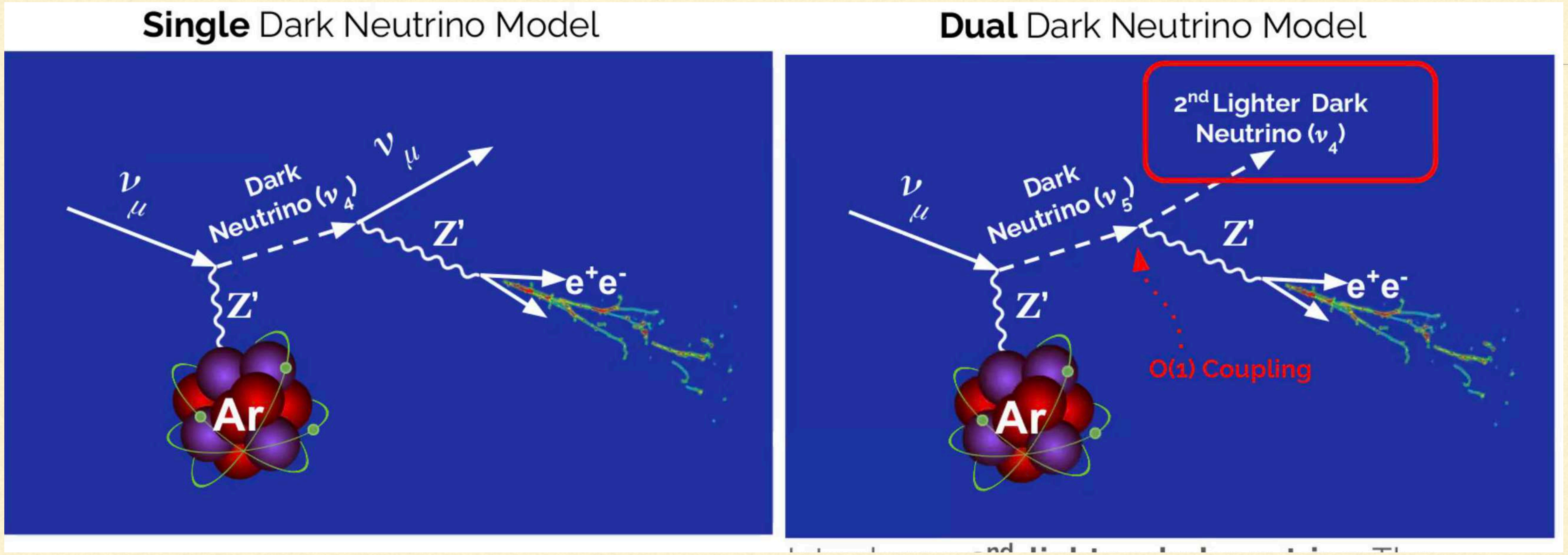
Single Dark Neutrino Model



Dual Dark Neutrino Model



MicroBooNE has conducted a broad search for these classes of dark neutrino models covering both light and heavy Z′ vector boson regimes, as well as both single and dual dark neutrino scenarios.



Single Dark Neutrino Model

Dual Dark Neutrino Model

[36] E. Bertuzzo, S. Jana, P. A. N. Machado, and R. Zukanovich Funchal, Phys. Rev. Lett. **121**, 241801 (2018), arXiv:1807.09877 [hep-ph].

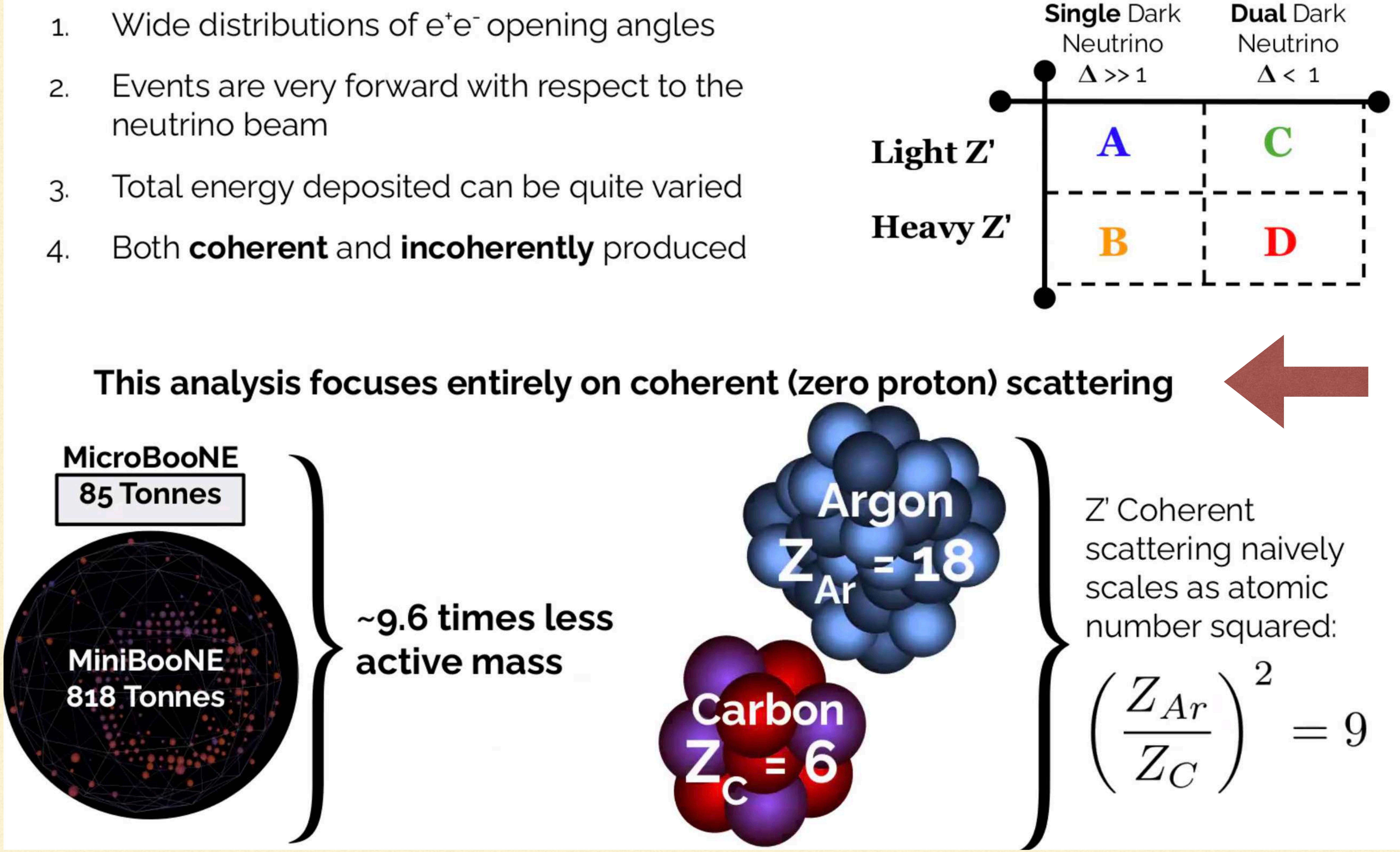
[37] P. Ballett, S. Pascoli, and M. Ross-Lonergan, Phys. Rev. D **99**, 071701 (2019), arXiv:1808.02915 [hep-ph].

[38] P. Ballett, M. Hostert, and S. Pascoli, Phys. Rev. D **101**, 115025 (2020), arXiv:1903.07589 [hep-ph].

[39] A. Abdullahi, M. Hostert, and S. Pascoli, Phys. Lett. B **820**, 136531 (2021), arXiv:2007.11813 [hep-ph].

[40] W. Abdallah, R. Gandhi, and S. Roy, JHEP **12**, 188 (2020), arXiv:2006.01948 [hep-ph].

Features of the signal for this study …….



1. Wide distributions of e^+e^- opening angles
2. Events are very forward with respect to the neutrino beam
3. Total energy deposited can be quite varied
4. Both **coherent** and **incoherently** produced

	Single Dark Neutrino $\Delta >> 1$	**Dual** Dark Neutrino $\Delta < 1$
Light Z'	A	C
Heavy Z'	B	D

This analysis focuses entirely on coherent (zero proton) scattering

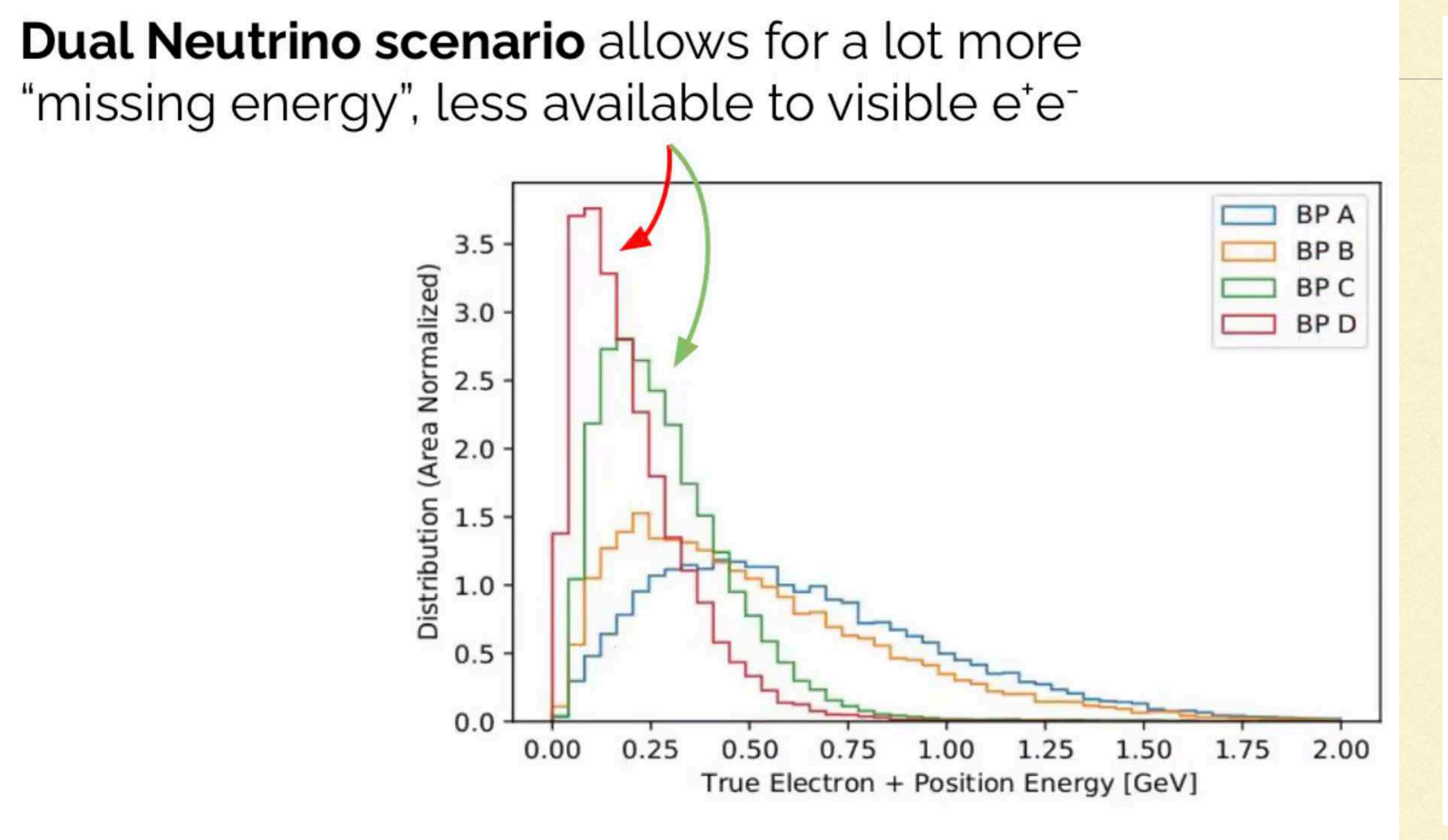
MicroBooNE
85 Tonnes

MiniBooNE
818 Tonnes

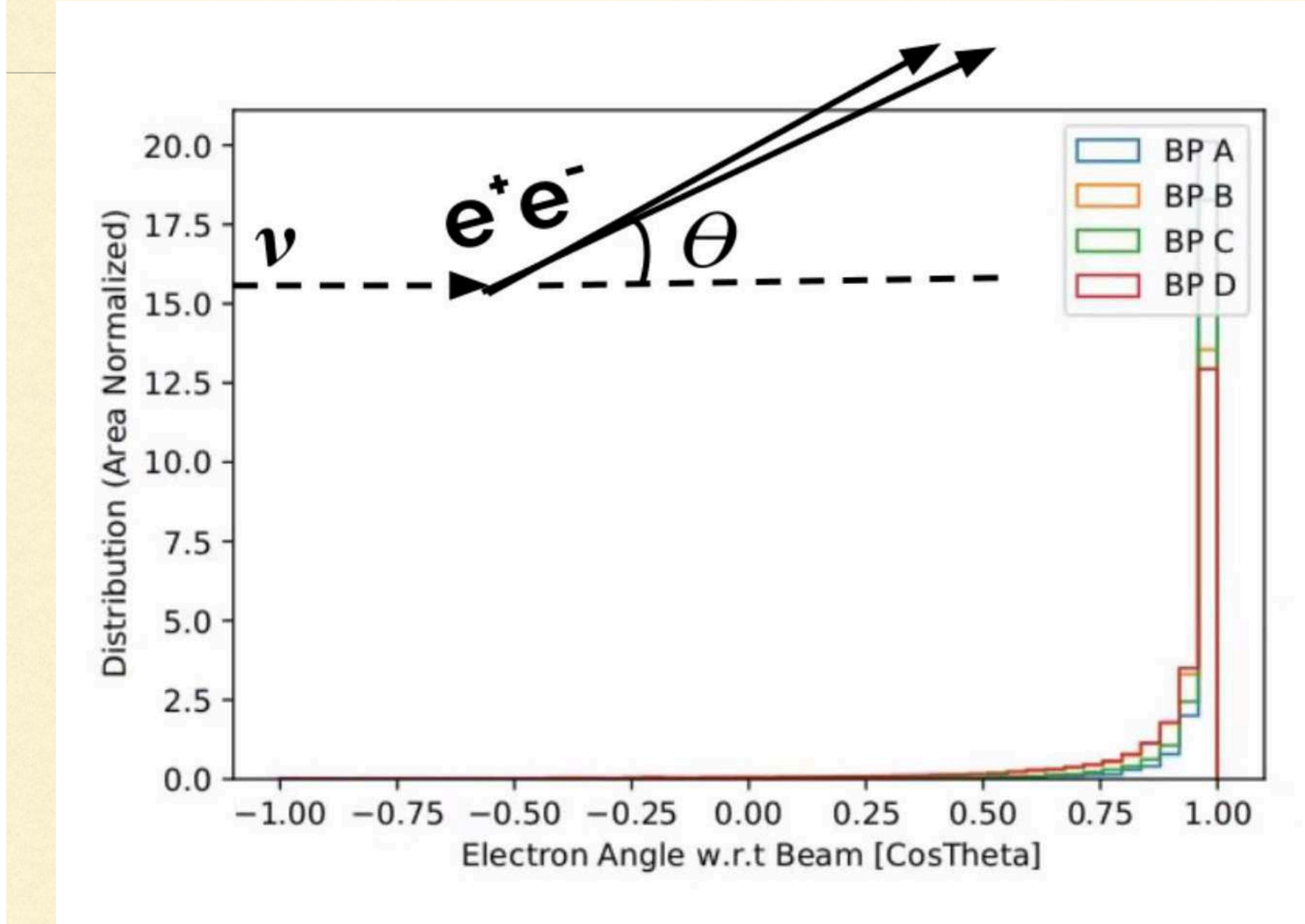
~9.6 times less active mass

Z' Coherent scattering naively scales as atomic number squared:

$$\left(\frac{Z_{Ar}}{Z_C}\right)^2 = 9$$



Dual Neutrino scenario allows for a lot more "missing energy", less available to visible e^+e^-



Note broad characteristics of expected signal for both coherent and incoherent events combined and heavy and light Z′.

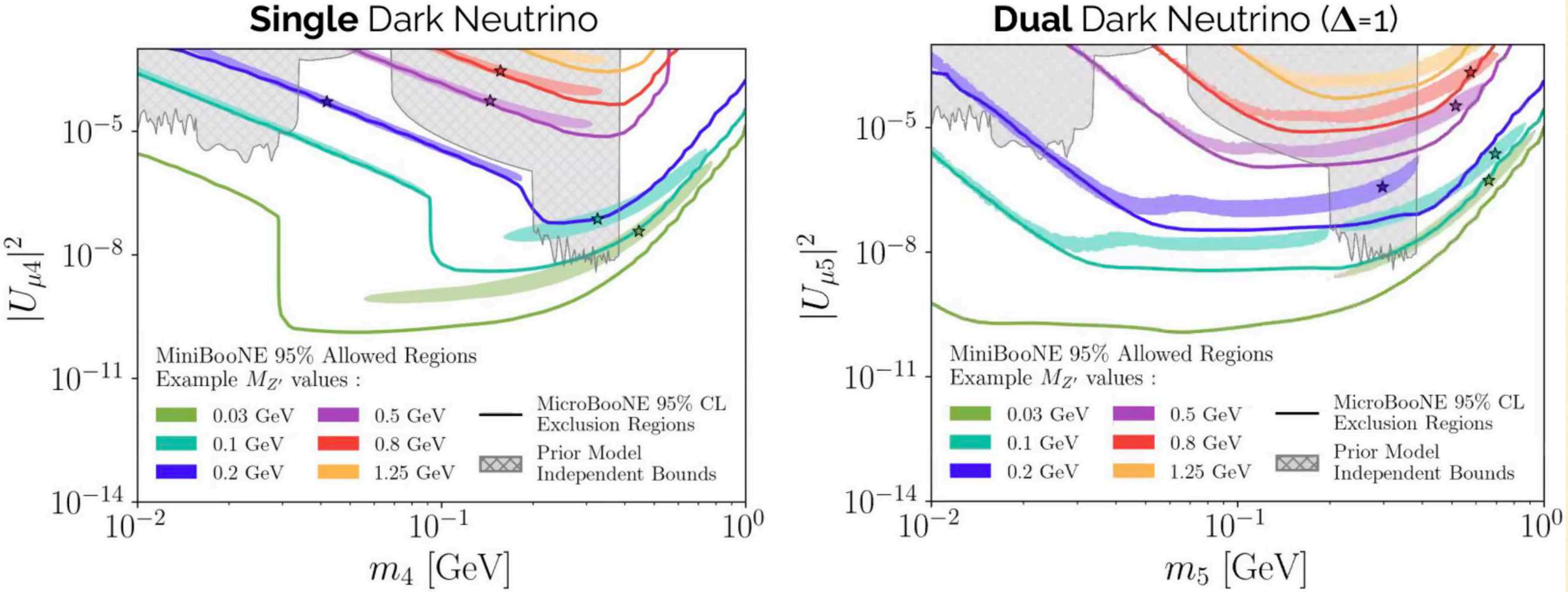
Significant number of events due to coherent scattering (zero proton, no nuclear activity)

Both low and high energy events in distribution, but more at low energies especially in dual dark neutrino scenario.

Significant number of events in forward or almost forward directions

Models with light or heavy vector mediators and HNLs are now highly constrained by the current measurements of coherent events by MicroBooNE

A typical example of the coherent vs incoherent distribution in a vector model



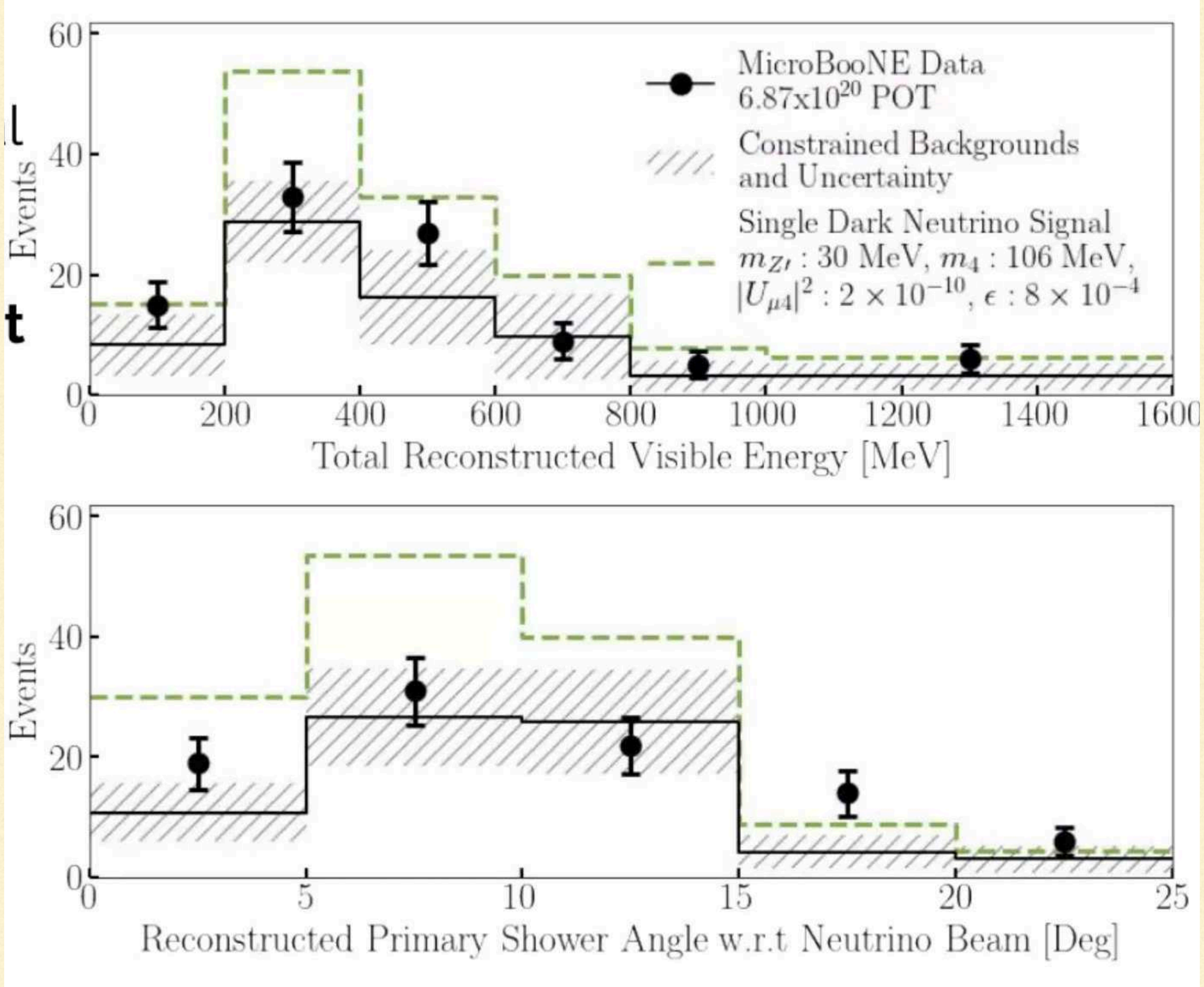
The world's first direct limits on these dark sector models and, at the 95% confidence level, **excludes the majority of the parameter space viable as a solution to the MiniBooNE anomaly** (especially for light Z' boson masses)

Summary of MicroBooNE e+e+ via Z' results

MicroBooNE observes no evidence for a coherent e'e signal consistent with a single or dual, dark neutrino scenario.

Subsequently they place the first direct bounds on this class of dark sector models

At 95% C.L. exclude the majority of the model phase space motivated by MB anomaly



So what remains ?

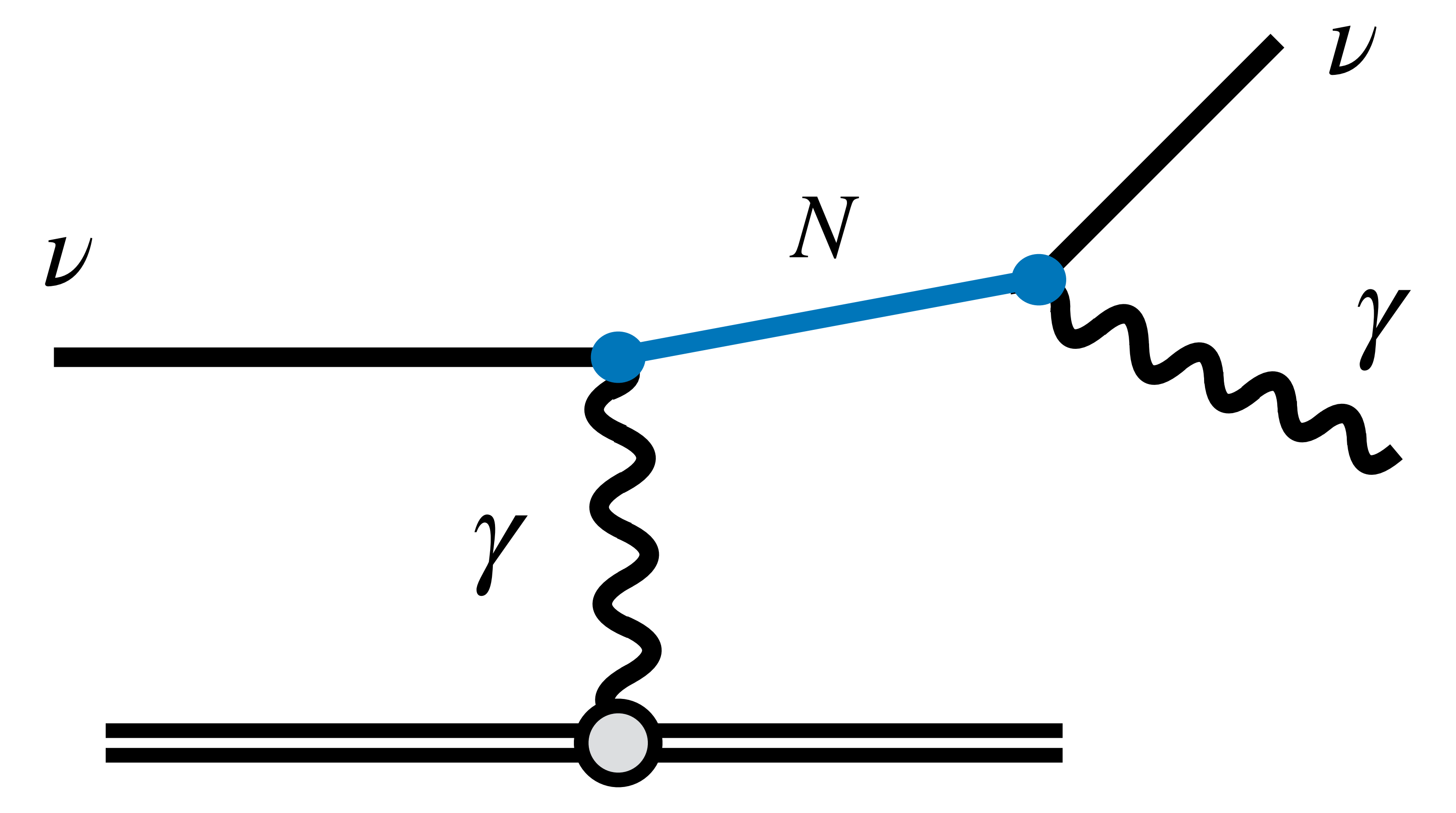
"Alternative dark sector models with different mediators, such as scalar mediators or those dominated by incoherent scattering, are not constrained by this result and remain exciting avenues for future exploration."

MicroBooNE Collaboration,
2502.10900

Mark Ross-Lonergan on behalf of the MicroBooNE Collaboration, Fermilab W&C seminar Feb 15, 2025

- Definitive tests of "remaining" models can only come from experiment
- Some general statements about them, using what we know so far, can however be made.

A large number of proposals use the signal of a single photon and neutrino produced by an HNL decay, via a transition magnetic moment (TMM)



$$\mathscr{L} \supset \frac{\mu_{\rm tr}^{\alpha}}{2} \overline{\nu}_{\alpha} \sigma^{\mu\nu} N_R F_{\mu\nu} + \text{ h.c.},$$

[74] S. N. Gninenko, Phys. Rev. Lett. **103**, 241802 (2009), arXiv:0902.3802 [hep-ph].

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[78] A. Radionov, Phys. Rev. D **88**, 015016 (2013), arXiv:1303.4587 [hep-ph].

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ph].

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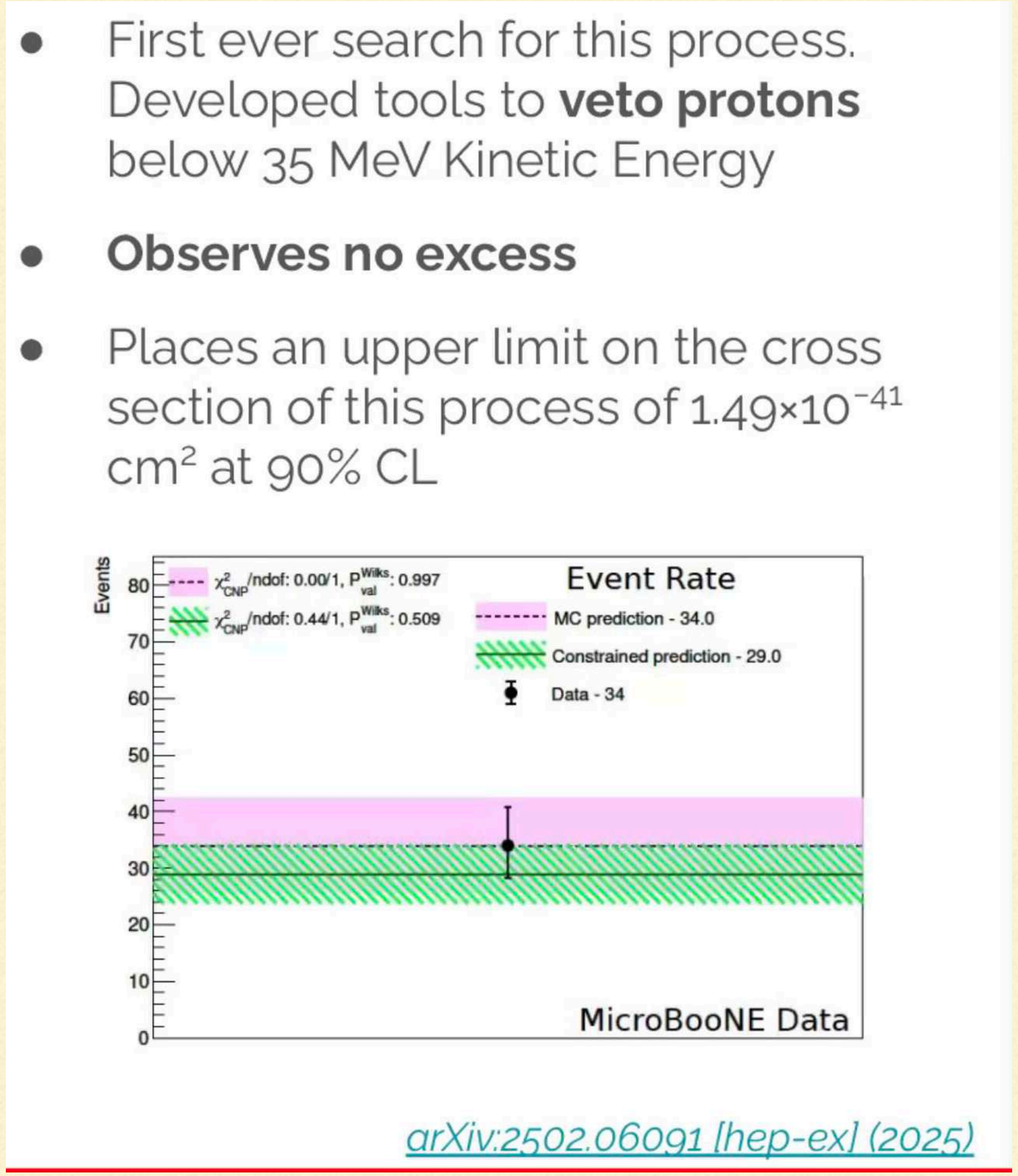
[83] N. W. Kamp, M. Hostert, A. Schneider, S. Vergani, C. A. Argüelles, J. M. Conrad, M. H. Shaevitz, and M. A. Uchida, Phys. Rev. D **107**, 055009 (2023), arXiv:2206.07100 [hep-ph].

[84] S. Bansal, G. Paz, A. Petrov, M. Tammaro, and J. Zupan, JHEP **05**, 142 (2023), arXiv:2210.05706 [hep-ph].

Testing TMM models……

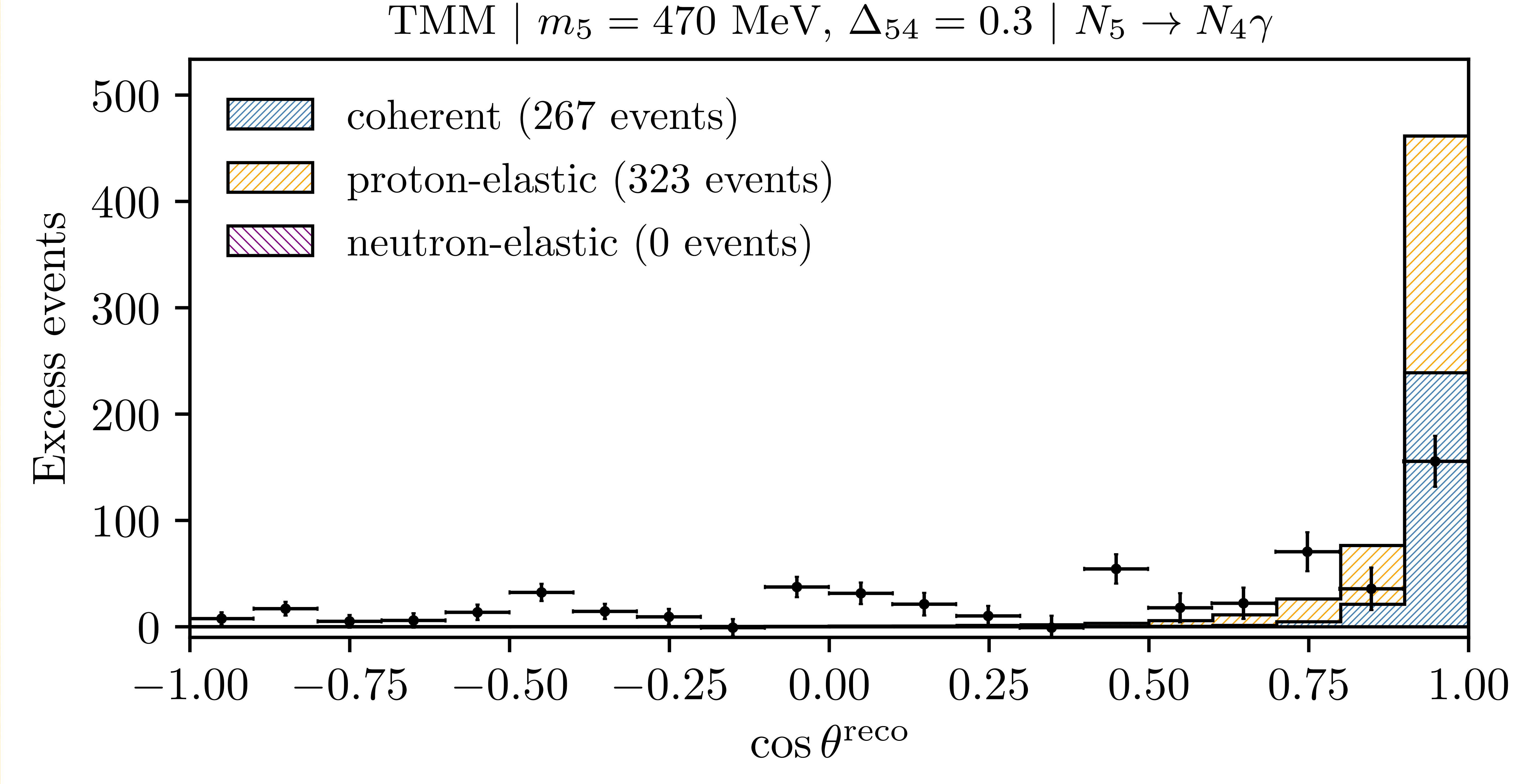


- First ever search for this process. Developed tools to **veto protons** below 35 MeV Kinetic Energy
- **Observes no excess**
- Places an upper limit on the cross section of this process of 1.49×10^{-41} cm^2 at 90% CL

arXiv:2502.06091 [hep-ex] (2025)

Coherent γ

Search for ***coherent γ***, an even rarer SM process than **Δ** Radiative.



Note significant percentage of coherent photon events at forward angles. (Not observed)

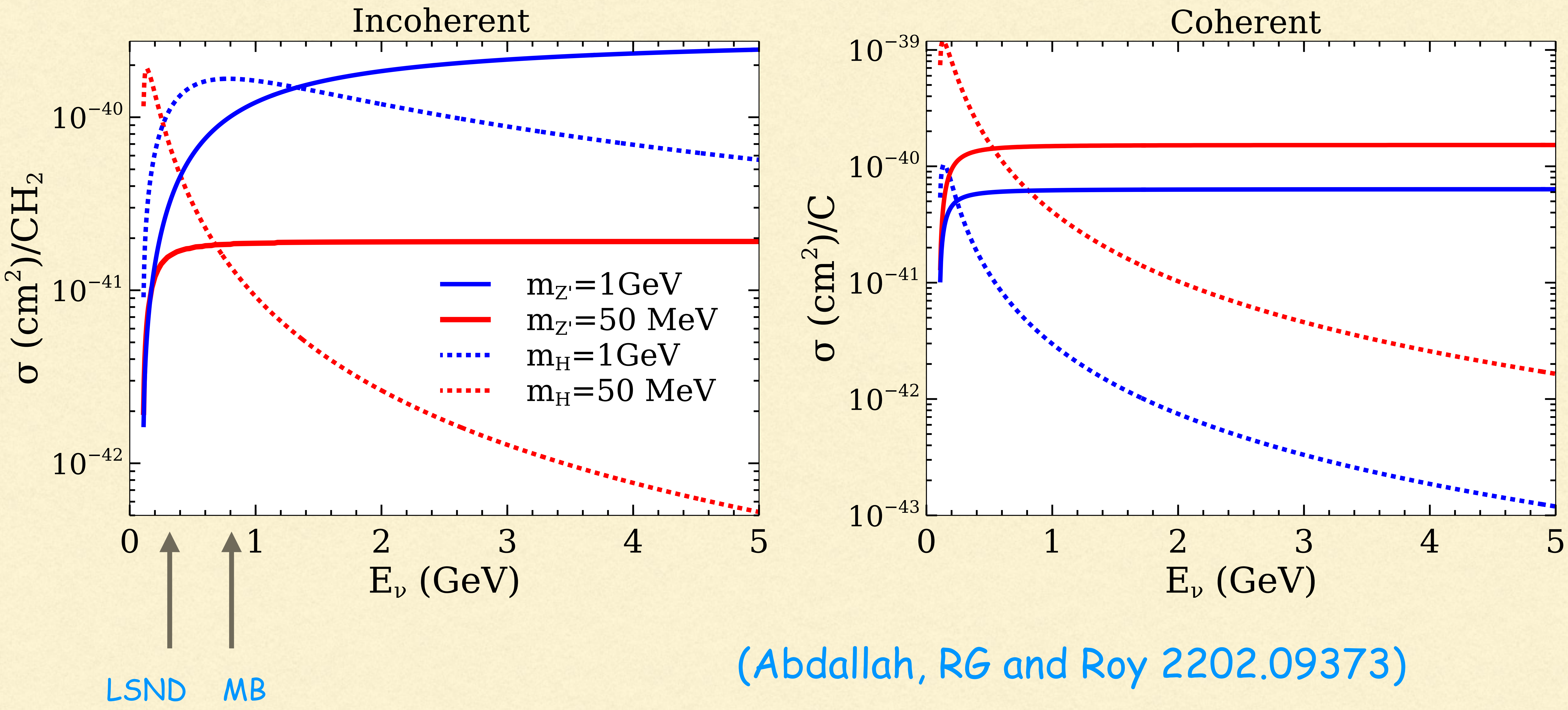
Need more data to come to definite conclusions

Using an additional Z′ and heavier sterile neutrinos, it is possible to get good fits to the MB data

Bertuzzo, Jana, Machado & Funchal, 1807.09877;
Ballet, Pascoli, Ross-Lonergon 1808.02915;
Abdallah, RG and Roy 2006.01948)

However, it is very difficult to explain both LSND and MB simultaneously using these ingredients, because a vector mediator does not give enough events at LSND

Additional information emerges if one requires the same new physics to explain both LSND and MB, ……



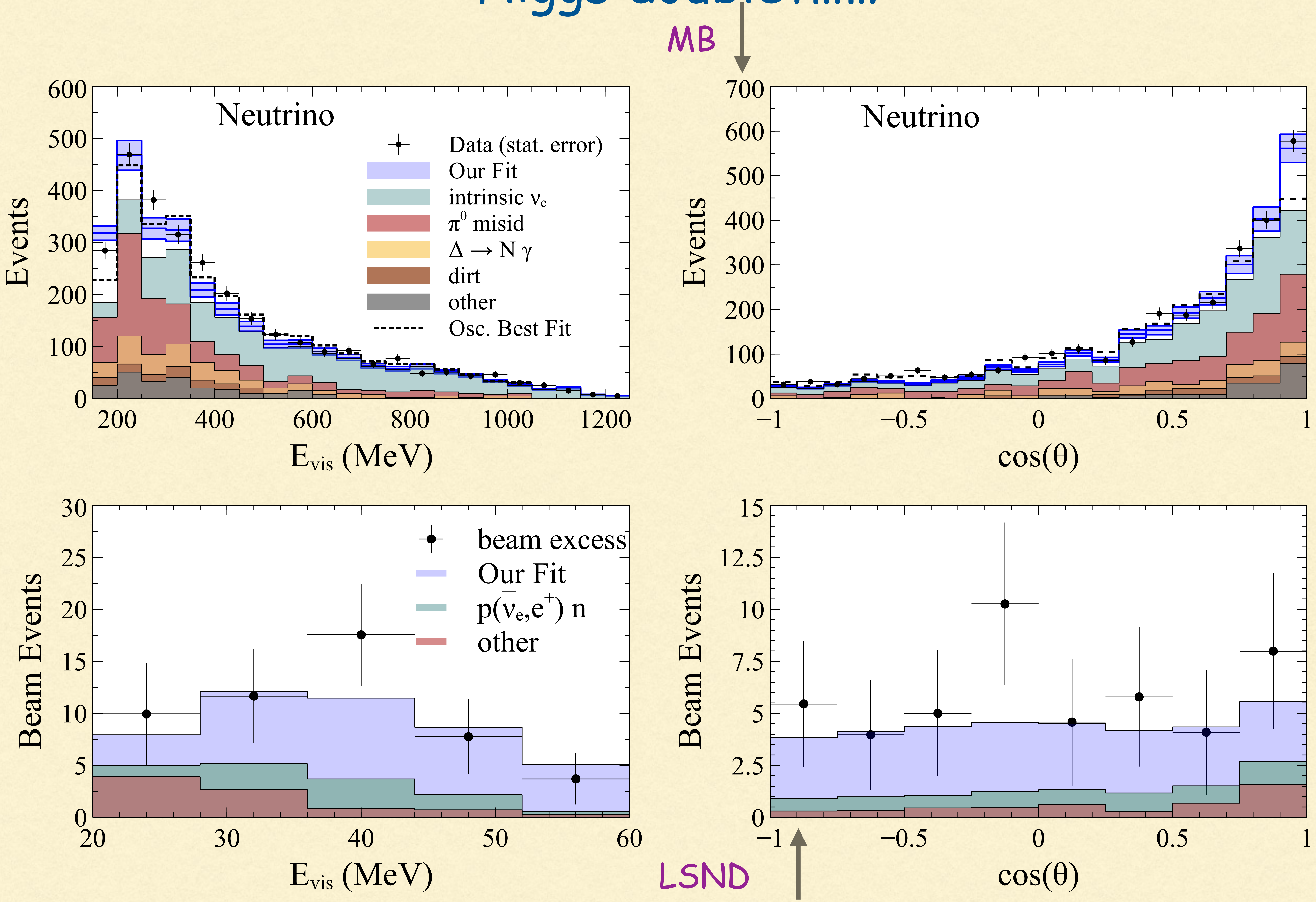
(Abdallah, RG and Roy 2202.09373)

Vector models, given the shape of the xsec, violate constraints by experiments with higher E, e.g. CHARM II (E_nu ~ 20 GeV and MINERvA, E_nu ~ 4-5 GeV)

Scalar mediators not only avoid HE constraints that vector mediators have difficulty avoiding, but also give enough events at LSND once you get the required number at MB.

Results with a light real scalar and an intermediate CP even Higgs from a second Higgs doublet……

(Abdallah, RG and Roy 2010.06159)



Real scalar + 2nd Doublet:

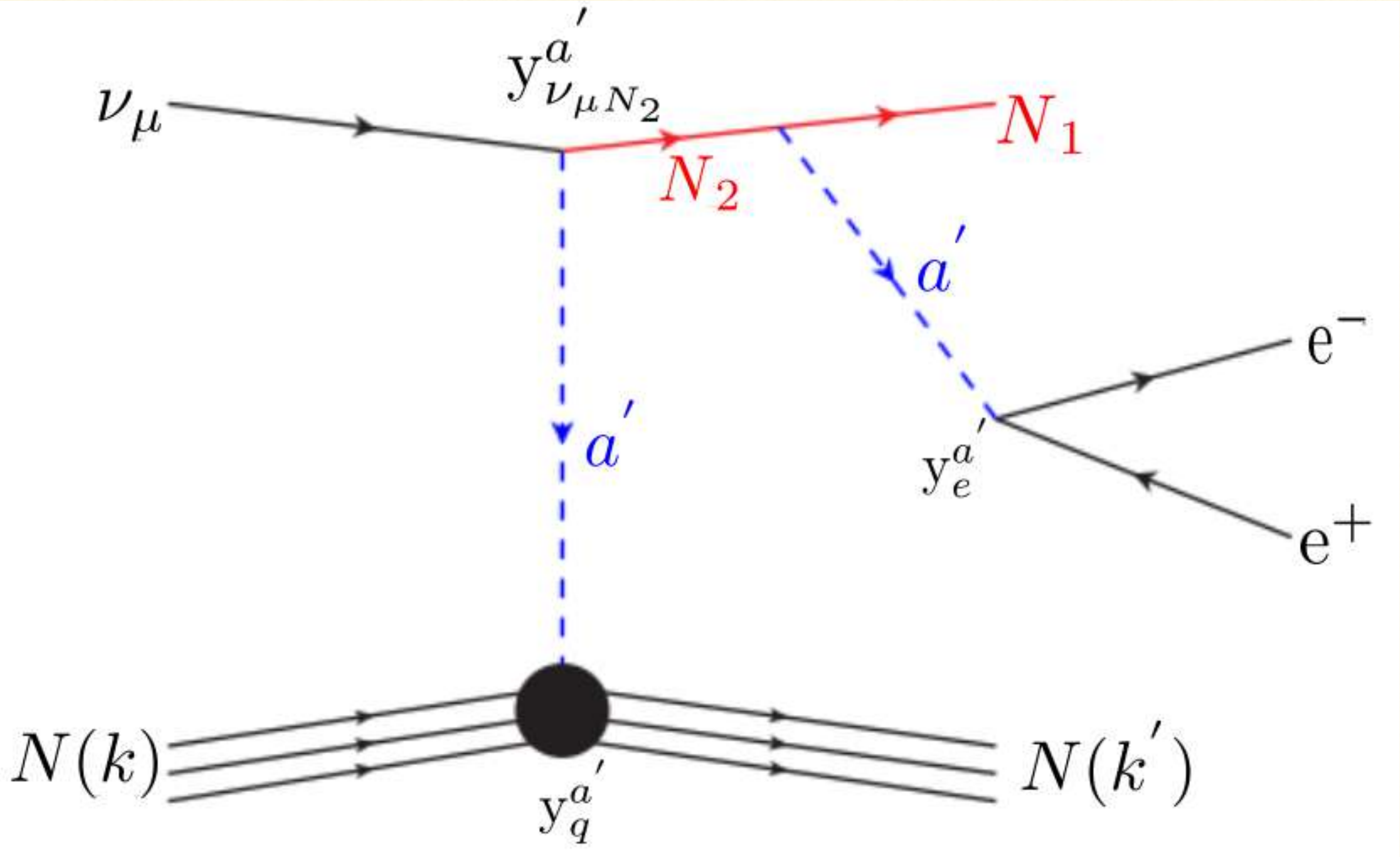
85% incoherent events

15% coherent events

Thus consistent with observations

Of lack of coherent events.

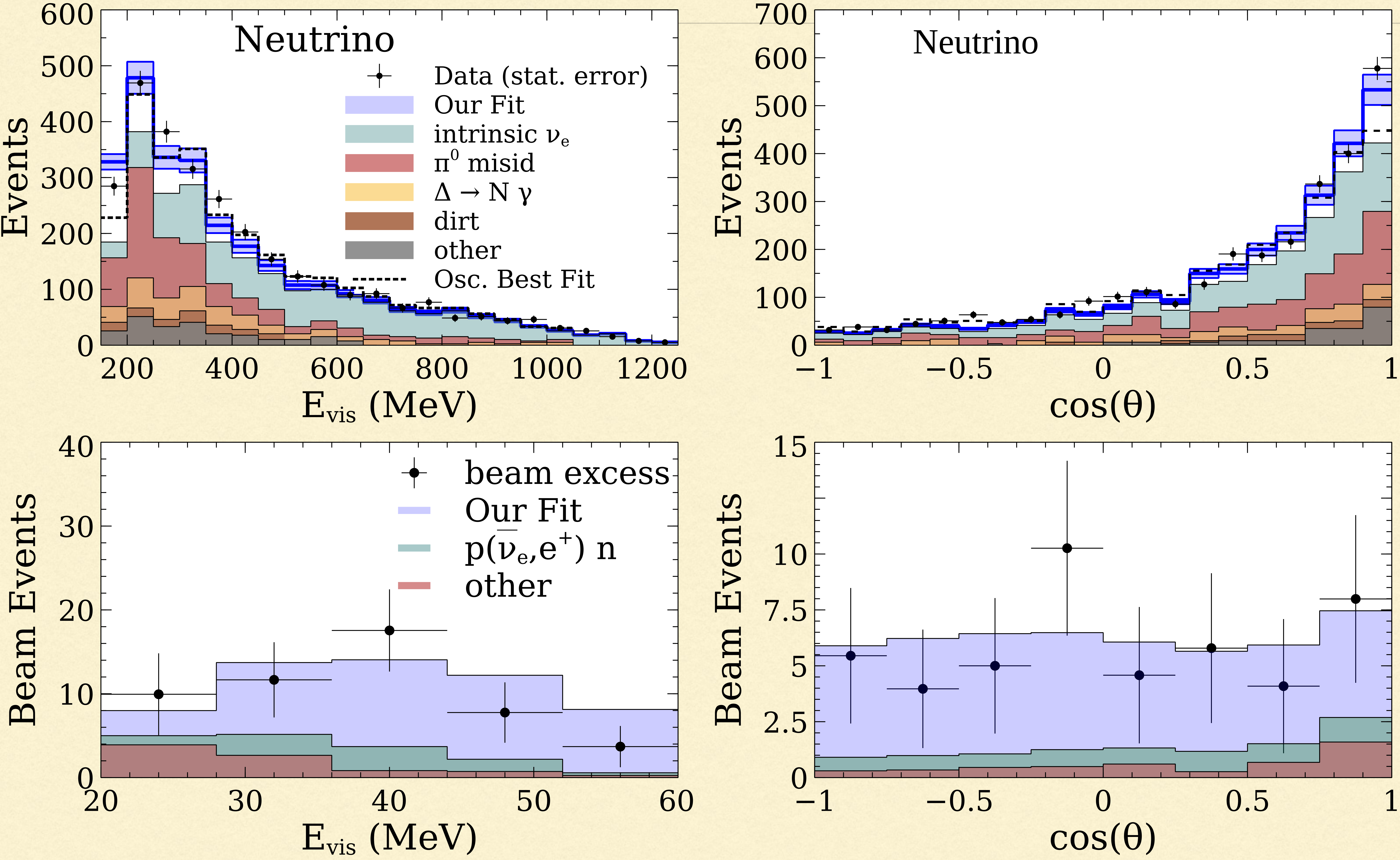
A pseudo scalar mediator……



arXiv 2406.07643 ;https://doi.org/10.1007/JHEP10(2024)086
W. Abdallah, RG, T. Ghosh, N. Khan, Samiran Roy, Subhojit Roy

We extend the scalar sector of the SM by incorporating a second Higgs doublet, and also add a singlet pseudoscalar $\phi_{h'} = i\,A_3^0/\sqrt{2}$. Additionally, three right-handed neutrinos help generate neutrino masses via the seesaw mechanism and participate in the interaction which generates electron-like signals in MB and LSND. We can write the scalar potential V as

$$V = V_{2\text{HDM}} + V_{h'}, \tag{2.1}$$



arXiv 2406.07643 ;https://doi.org/10.1007/JHEP10(2024)086
W. Abdallah,
RG, T. Ghosh,
N. Khan,
Samiran Roy,
Subhojit Roy

Pseudoscalar + 2nd HDoublet:

100% incoherent events due to spin dependent couplings

Thus consistent with observations

Of lack of coherent events.

Conclusions and Summary……

LSND and MB are persistent and statistically significant anomalies

Many checks over the years have failed to reveal SM physics explanations or systematic uncertainties as being responsible for the observed excesses

MicroBooNE has carried out extensive tests of some of both SM (e.g Delta production) and new physics explanations (e.g. sterile neutrinos

Its most recent results point to new physics interactions which may involve HNL, dark portals and and new interactions provided such interactions are dominated by incoherent scattering processes.

This has eliminated proposed solutions where new interactions are mediated by vectors. It also seems to disfavour solutions via transition magnetic moments, although more definitive data and experimental probing is necessary.

Scalar and pseudo scalar mediators lead to significant or total incoherent scattering and remain viable solutions and are able to account for both LSND and MB, as well as evade HE constraints.

Next set of MicroBooNE results and SBND results will provide possibly definitive answers to these puzzles.

> *"How often have I said to you that when you have eliminated the impossible, whatever remains, however improbable, must be the truth?*

Sherlock Holmes in *The Sign of the Four*

(Stay tuned into this fascinating physics detective story)………

Thank you for your attention!

The interaction and the model……

$$V_{\text{2HDM}} = \mu_1|\phi_h|^2 + \mu_2|\phi_H|^2 + \frac{\lambda_1}{2}|\phi_h|^4 + \frac{\lambda_2}{2}|\phi_H|^4 + \lambda_3|\phi_H|^2|\phi_h|^2 + \lambda_4(\phi_h^\dagger\phi_H)(\phi_H^\dagger\phi_h)$$
$$+\frac{\lambda_5}{2}\{(\phi_h^\dagger\phi_H)^2 + h.c\} + (\lambda_6|\phi_h|^2 + \lambda_7|\phi_H|^2)\,(\phi_h^\dagger\phi_H + \phi_H^\dagger\phi_h),$$

$$V_{h'} = \mu'|\phi_{h'}|^2 + \lambda_2'|\phi_{h'}|^4 + \lambda_3'|\phi_h|^2|\phi_{h'}|^2 + \lambda_4'|\phi_H|^2|\phi_{h'}|^2 + \{(\lambda_5'|\phi_{h'}|^2 - \mu_3)(\phi_h^\dagger\phi_H)$$
$$+(m_1|\phi_h|^2 + m_2|\phi_H|^2 + m_3\phi_h^\dagger\phi_H - m_s\phi_{h'})\phi_{h'} + h.c.\}.$$

$$\phi_h = \begin{pmatrix} G^+ \\ \frac{v+H_1^0+iG^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_H = \begin{pmatrix} H_2^+ \\ \frac{H_2^0+iA_2^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_{h'} = i\,A_3^0/\sqrt{2}\,.$$

$$\langle\phi_h\rangle = v(\equiv v_{SM}) \simeq 246 \text{ GeV}$$

arXiv 2406.07643 ;https://doi.org/10.1007/JHEP10(2024)086
W. Abdallah, RG, T. Ghosh, N. Khan, Samiran Roy, Subhojit Roy

While the combination of a light (15-20 MeV) scalar and an intermediate (750 MeV) one provide a very good fit to MB and LSND, a light pseudo scalar of the same mass (17 MeV) does better

This is because it only has incoherent scattering with the nucleons of the spin-0 Carbon nucleus hence the event contribution is not just predominantly forward.

The important a' couplings for our purpose are those with quarks and electrons

$$\mathcal{L}_{a'qq} = y_q^{a'}\, a'\bar{q}\, i\gamma_5\, q\,.$$

Effective couplings to nucleons can then be calculated

$$F_N = \frac{m_N}{m_q} \sum_{q=u,d,s} \Delta_q^{(N)} \left(y_q^{a'} - \sum_{q'=u,..,t} y_q^{a'} \frac{\overline{m}}{m_{q'}} \right), \tag{3.2}$$

where $\Delta_q^{(N)}$ are the quark spin components of the nucleon N,

$$\frac{1}{\overline{m}} = \frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s}, \tag{3.3}$$

$\Delta_u^{(p)} = 0.84$, $\Delta_d^{(p)} = -0.44$, $\Delta_s^{(p)} = -0.03$, $\Delta_u^{(n)} = -0.44$, $\Delta_d^{(n)} = 0.84$, $\Delta_s^{(n)} = -0.03$ [88].

The total sec is given by

$$\left[\frac{d\sigma}{dE_{N_2}}\right]_{\mathrm{CH}_2} = \Big[\underbrace{(8F_p^2+6F_n^2)}_{\text{incoherent}}\Big]\frac{d\sigma}{dE_{N_2}}.$$

Total events

$$\mathrm{N}_{\text{events}} = \eta\int dE_\nu dE_{N_2}\frac{d\Phi^\nu}{dE_\nu}\frac{d\sigma}{dE_{N_2}}\times \mathrm{BR}(N_2\to N_1 a'),$$

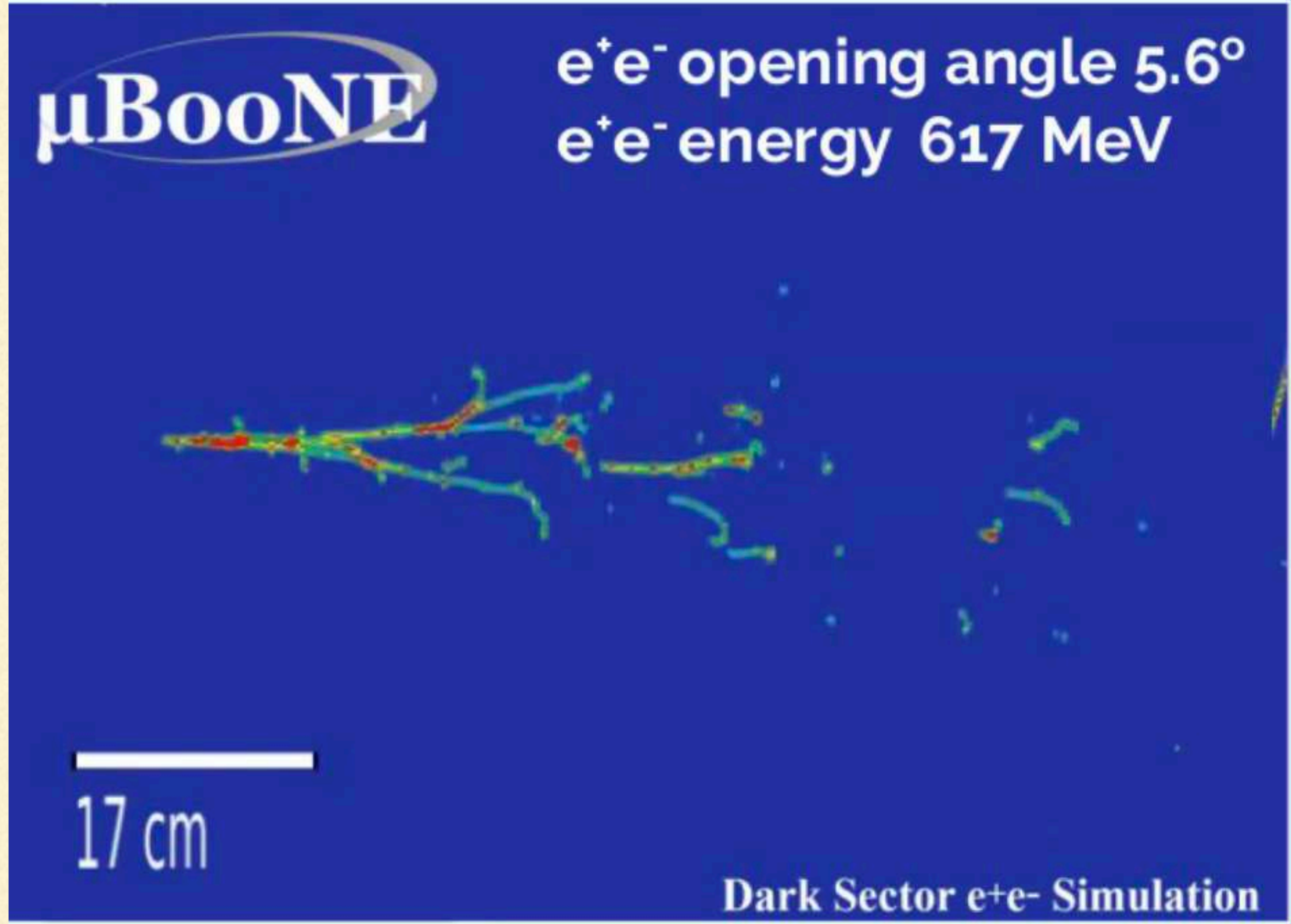
m_{N_1}	m_{N_2}	m_{N_3}	$y_u^{a'}\times 10^6$	$y_e^{a'}\times 10^5$	$y_\mu^{a'}\times 10^5$	$M_{H^\pm}$	$y_c^{a'}$	$y_t^{a'}$
70 MeV	120 MeV	10 GeV	4.34	2.3	1	305 GeV	0	0

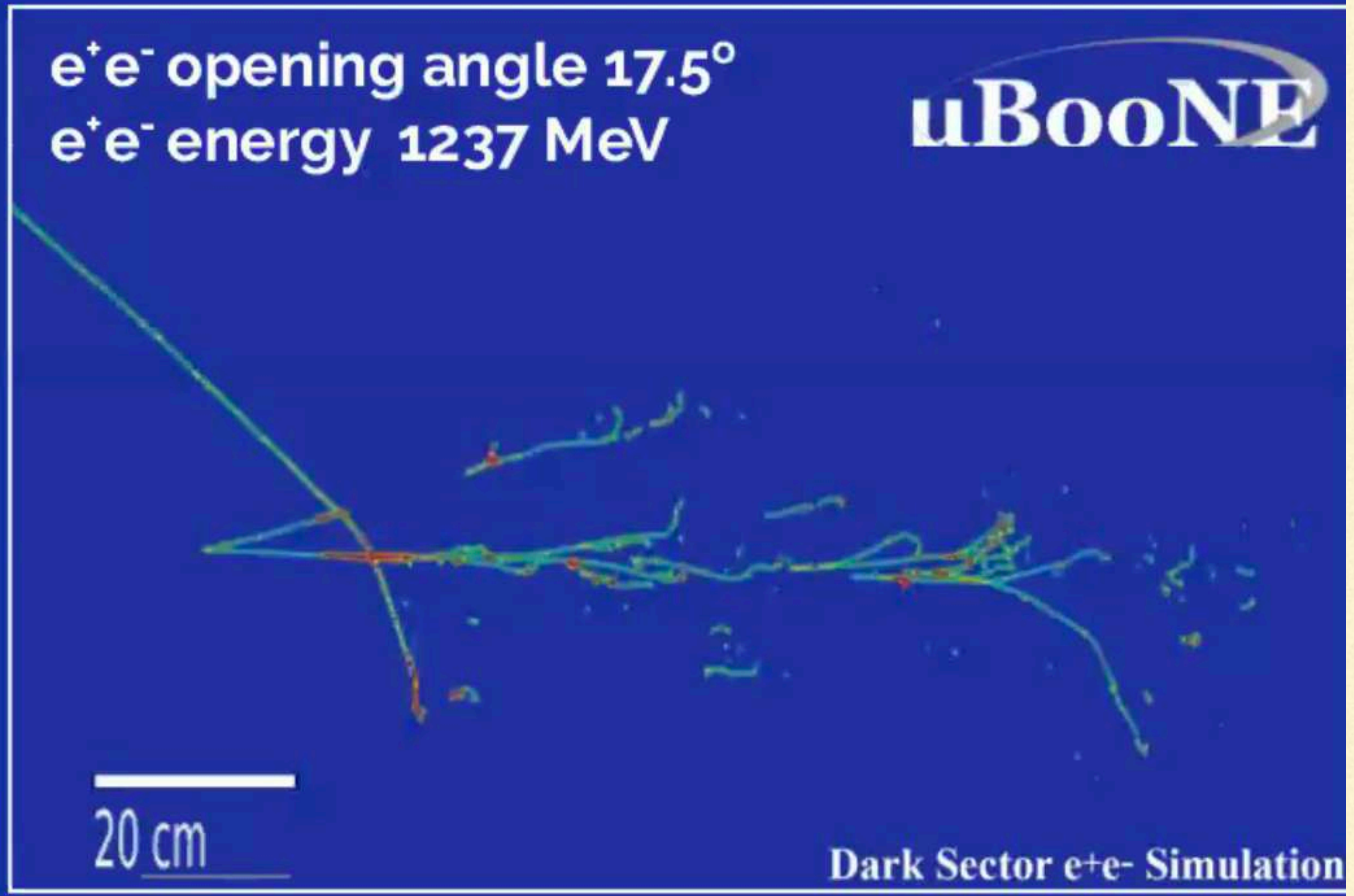
$M_{a'}$	M_H	$\sin\xi$	$y_d^{a'}\times 10^6$	$y_{\nu_\mu N_2}^{a'}\times 10^2$	$\lambda_{N_{12}}^{a'}$	M_A	$y_s^{a'}$	$y_b^{a'}$
17 MeV	300 GeV	0.01	4.0	3.15	0.1	400 GeV	0	0

Table 1: Benchmark parameter values used to generate the event spectrum in LSND and MB.

arXiv 2406.07643 ;https://doi.org/10.1007/JHEP10(2024)086
W. Abdallah, RG, T. Ghosh, N. Khan, Samiran Roy, Subhojit Roy

What does our signal look like in our detector?





1s0t

- Typical topology used in searching for SM single-photons
- Most sensitive to e^+e^- pairs with small opening angles

1s1t

- Lower energy electrons often reconstructed as tracks
- Track and shower reconstructed at same vertex, no conversion distance

2s0t

- Two clear distinct showers starting at same vertex is possible for higher energy, wider opening angle e^+e^- pairs

Tension between appearance and disappearance for active-sterile oscillations

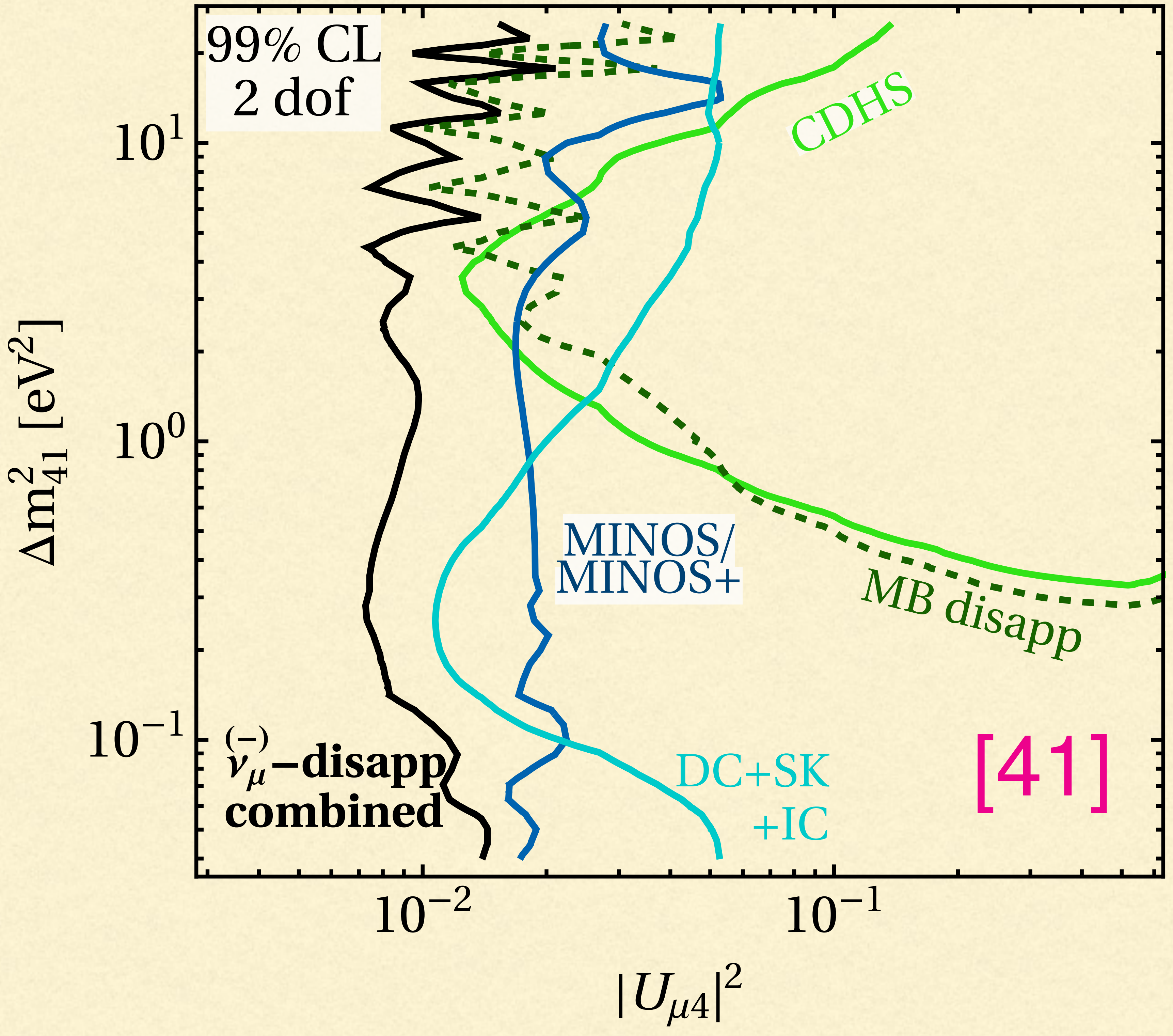
We note that non-zero nu_mu-nu_ e appearance requires both nu_e and nu_mu disappearance

Many experiments have looked for ν_μ disappearance :

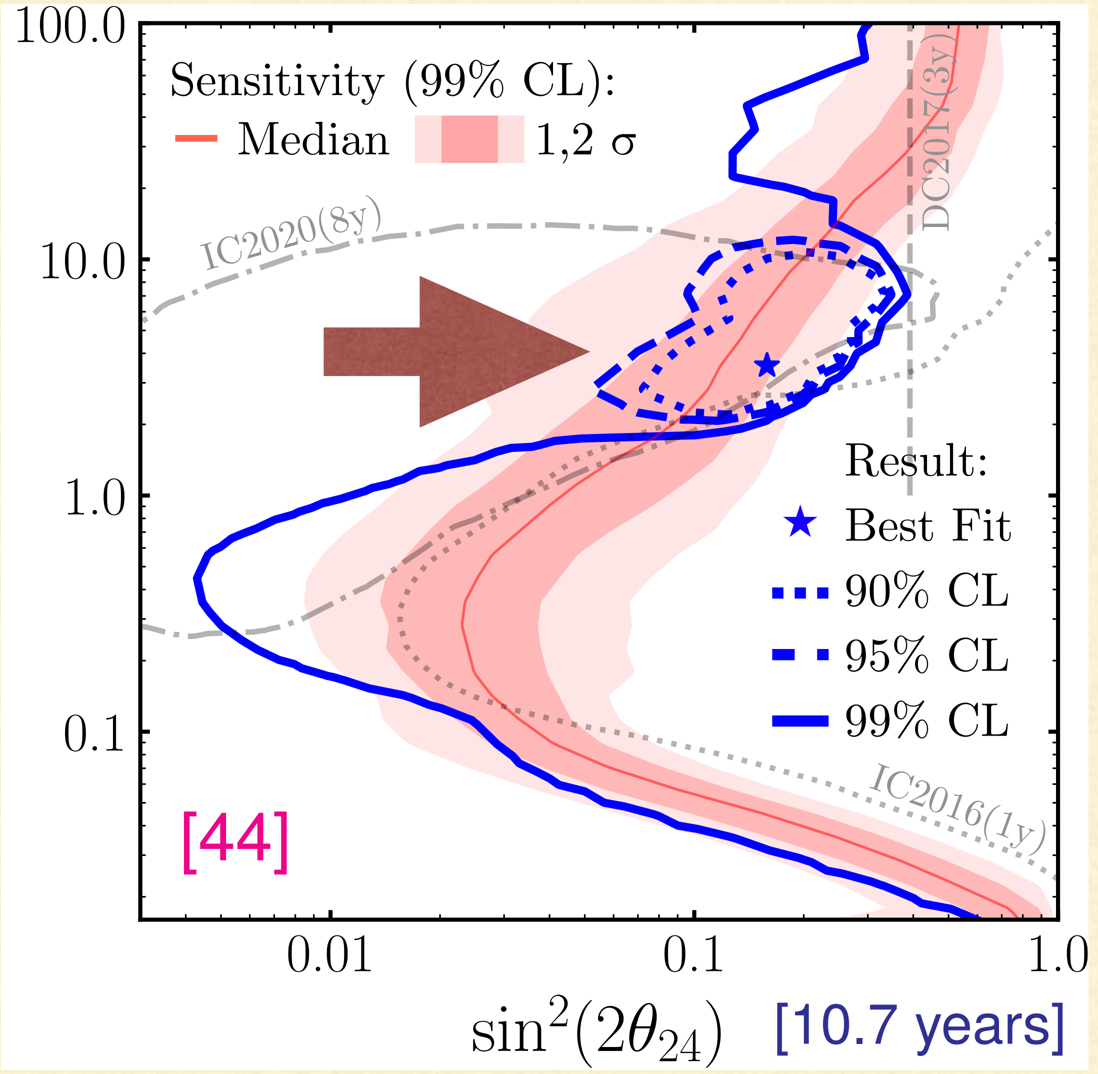
– CDHS (ν) – MiniBooNE (ν, $\bar{\nu}$) – SciBooNE (ν, $\bar{\nu}$)

– MINOS (ν) – NOνA (ν) – SK atmos (ν, $\bar{\nu}$)

- no hint of ν_μ disappearance has been observed;



Small island earlier allowed by IceCube is now disfavoured by their latest results, which require $|U_{\mu4}|^2{}_{mu4^2} < 0.0534$ at 90% CL under the assumption that $\Delta m^2 41 \geq$ 1eV^2.

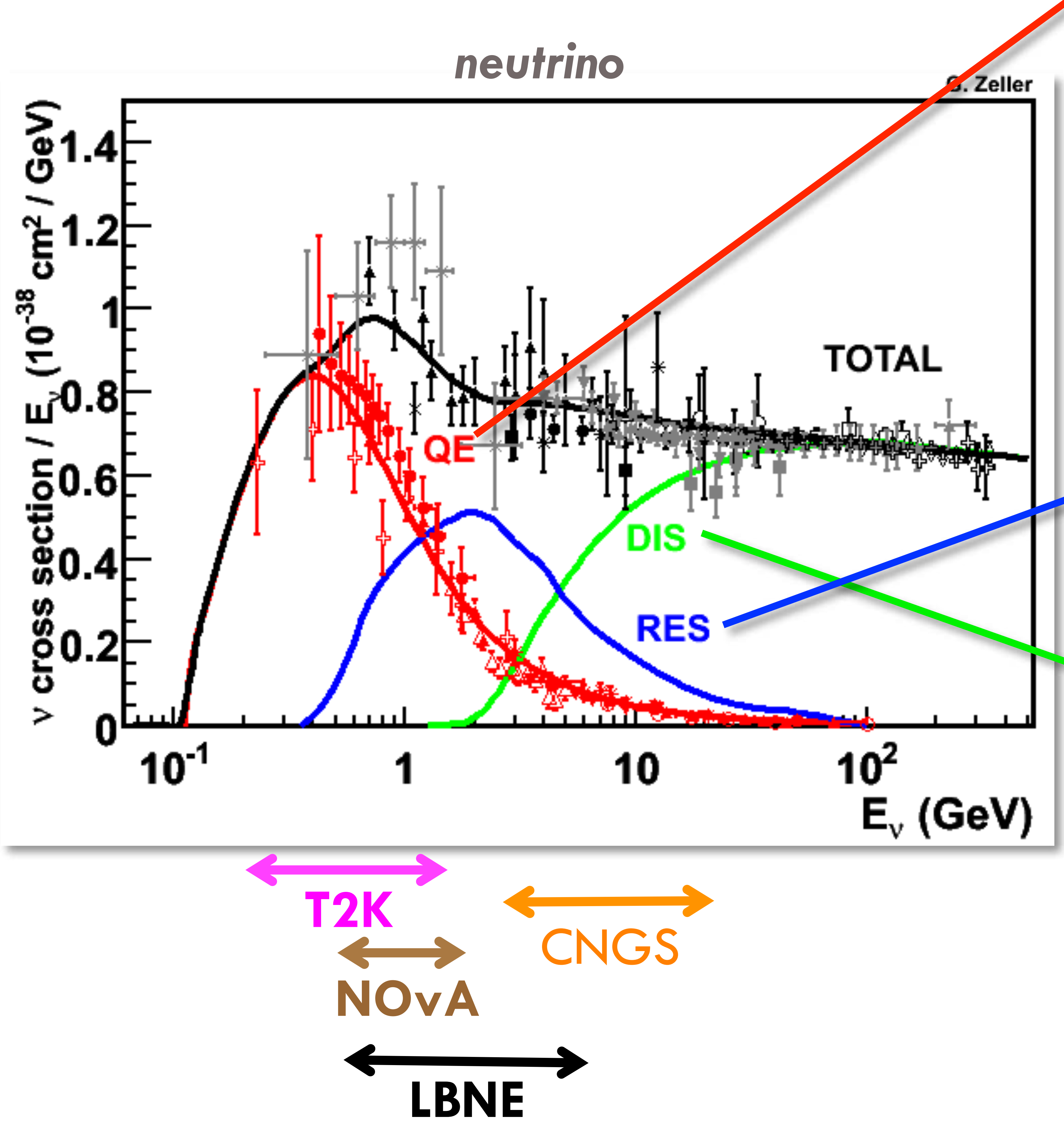


R. Abbasi et al, (IceCube Collab.) 2407.01314

Dentler et al, 1803.10661

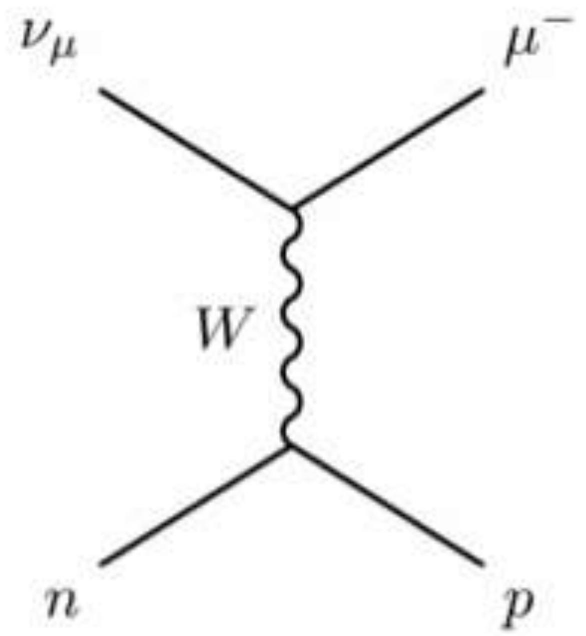
Complicated Region

(our accelerator-based ν event samples contain contributions from multiple reaction mechanisms)



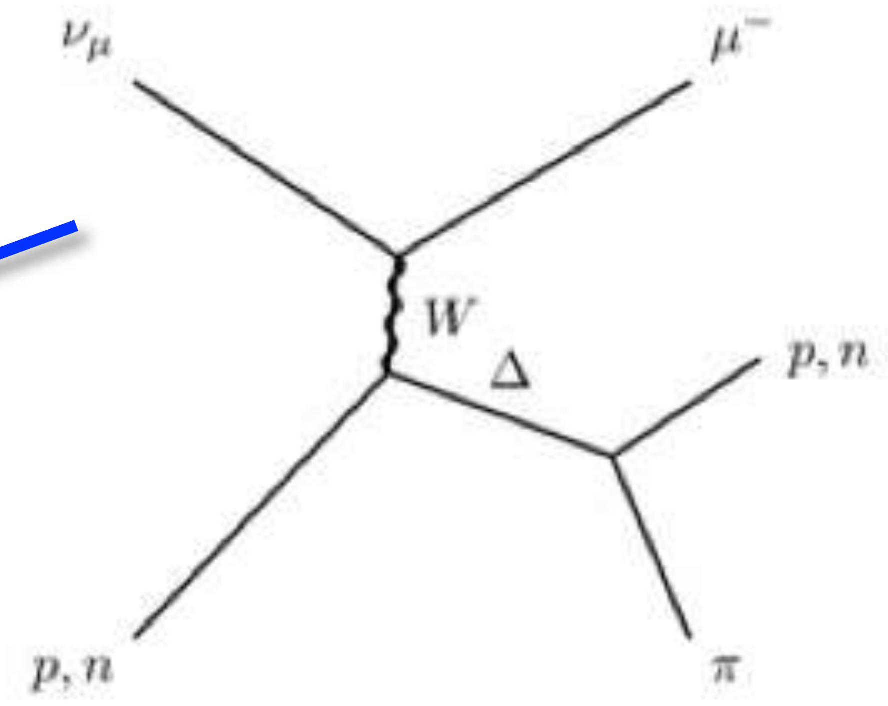
CC Quasi-elastic

nucleon changes,
but doesn't break up



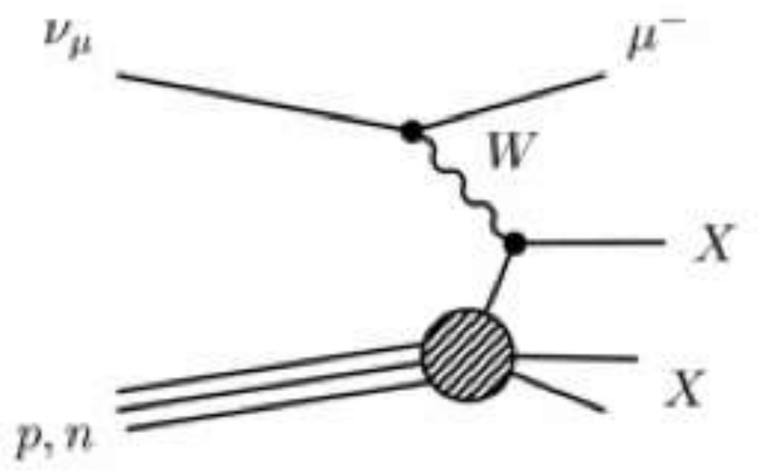
CC Single pion

nucleon excites to
resonance state



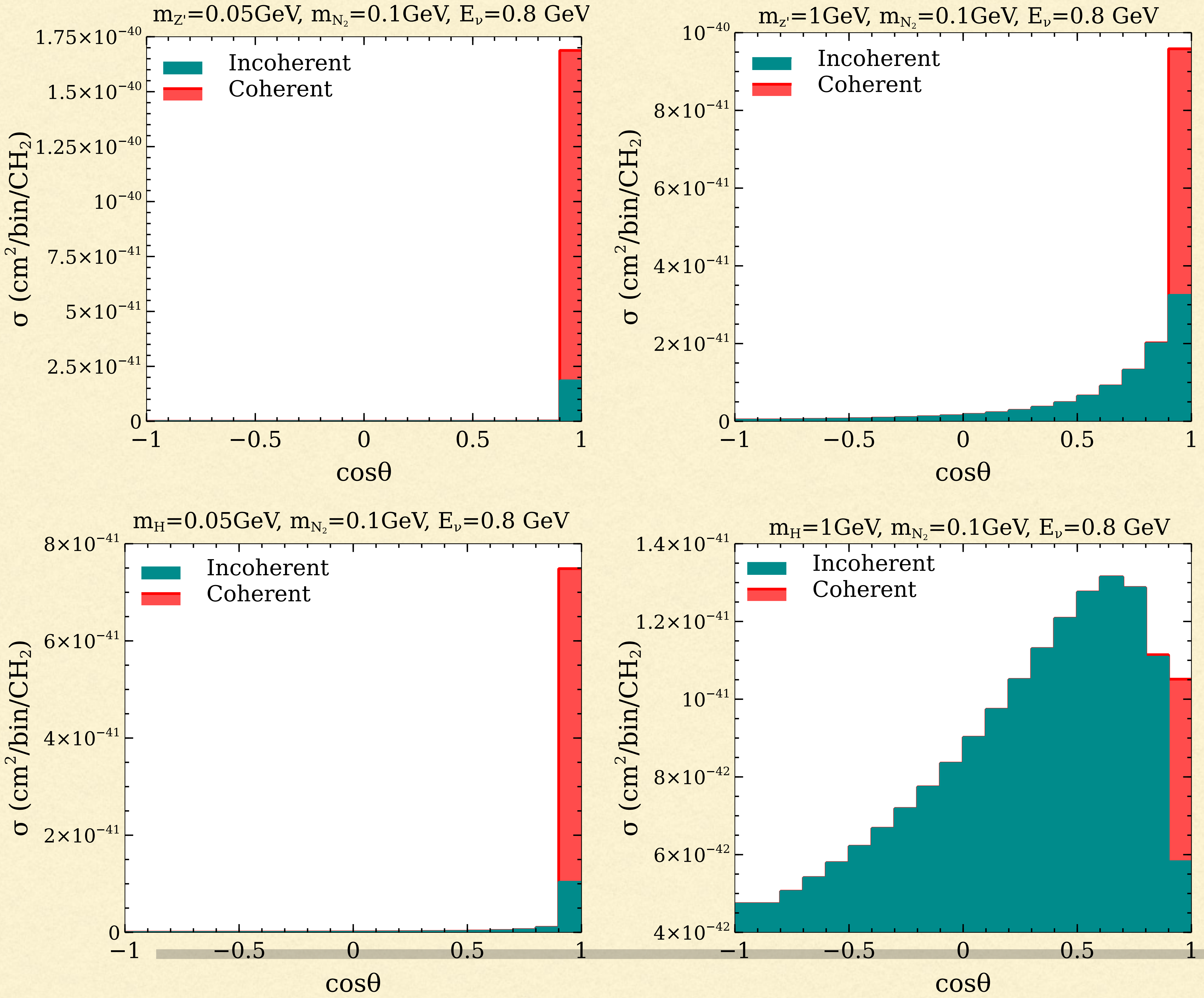
CC Deep Inelastic

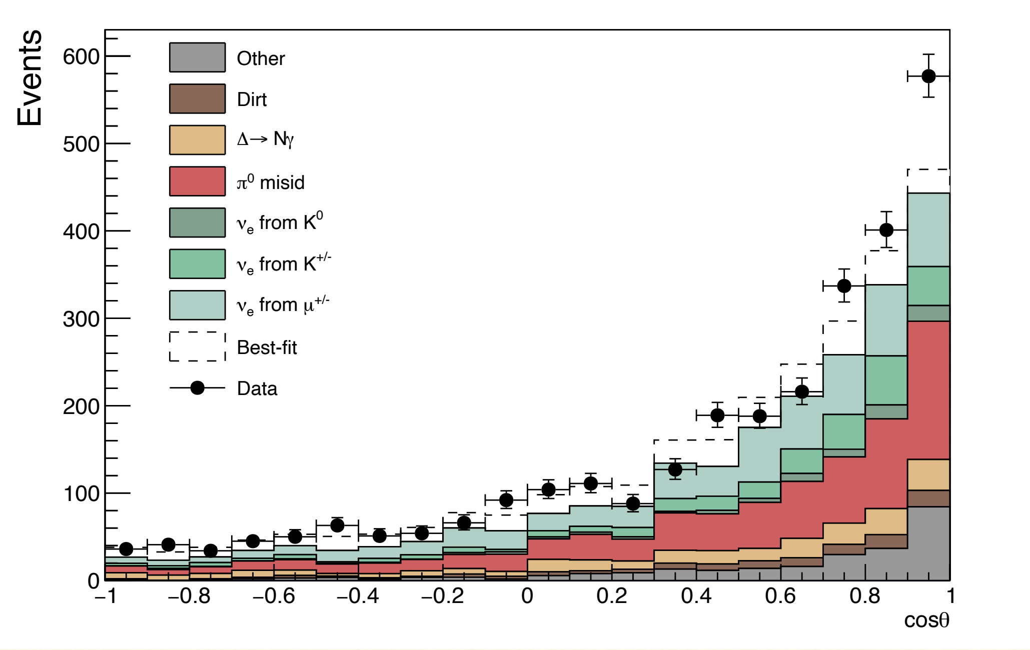
nucleon breaks up



let's start
with QE …

What does one learn if one demands that the new physics resolve both LSND and MB, as opposed to just MB.





By studying the angular distribution at MB for both light and not so light scalar and vector mediators, one discerns the need for both a light and an intermediate mass mediator

An intermediate mass scalar mediator tends to give event contributions to all angular bins, unlike a vector.

(Abdallah, RG and Roy 2202.09373)

MicroBooNE results……

"These results disfavor the hypothesis that the MiniBooNE low-energy excess originates solely from an excess of νe interactions. Instead, one or more additional mechanisms [45–52] are required to explain the MiniBooNE observations. "

(MicroBooNE Collab, 2210.10216)

- [45] A.de Gouvˆea,O.L.G.Peres,S.Prakash,andG.V. Stenico, arXiv:1911.01447 [hep-ph].
  (Sterile to active decay)
- [46] S. Vergani, N. W. Kamp, A. Diaz, C. A. Argu¨elles, J. M. Conrad, M. H. Shaevitz, arXiv:2105.06470 [hep-ph].
  (Mix of sterile osc and decay to active)
- [47] J. Asaadi, E. Church, R. Guenette, B. J. P. Jones, and A. M. Szelc, arXiv:1712.08019 [hep-ph].
  (New matter resonance effects)
- [48] D. S. M. Alves, W. C. Louis, and P. G. deNiverville, arXiv:2201.00876 [hep-ph].
  (New matter resonance effects)
- [49] E. Bertuzzo, S. Jana, P. A. N. Machado, and R. Zukanovich Funchal, arXiv:1807.09877 [hep-ph].
  (Up-scattering and additional Z′)
- [50] P. Ballett, S. Pascoli, and M. Ross-Lonergan, arXiv:1808.02915 [hep- ph].
  (Up-scattering and additional Z′)
- [51] W. Abdallah, R. Gandhi, and S. Roy, arXiv:2010.06159 [hep-ph].
  (Up-scattering and additional Z′)
- [52] W. Abdallah, R. Gandhi, and S. Roy, arXiv:2006.01948 [hep-ph].
  (Up-scattering and Additional scalars)

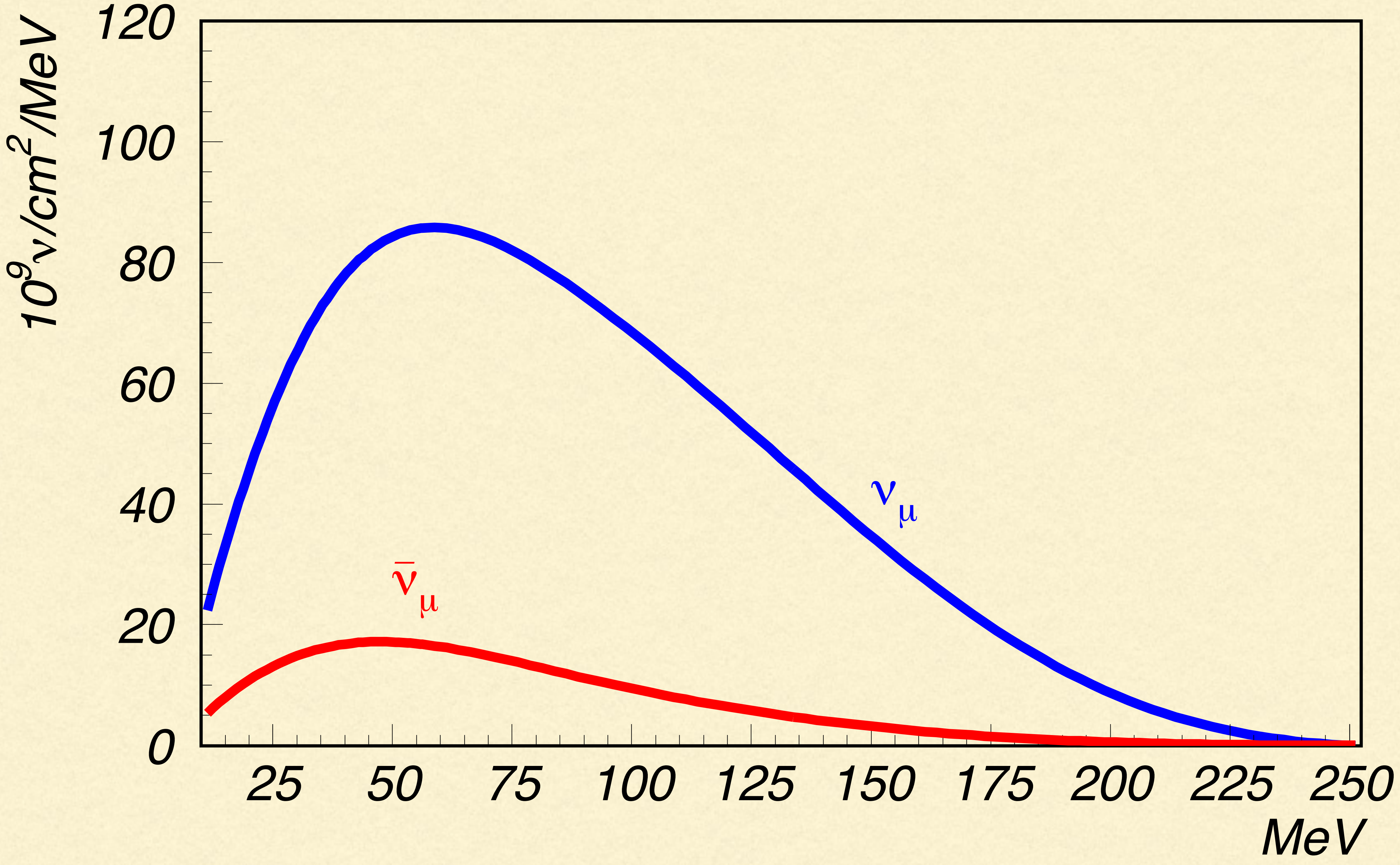
arXiv 2406.07643 ;https://doi.org/10.1007/JHEP10(2024)086
W. Abdallah, RG, T. Ghosh, N. Khan, Samiran Roy, Subhojit Roy

Remarks on LSND

Our model requires the production of a relatively heavy N_2 (120MeV).

$\bar{\nu}_\mu$ Flux from DAR is not energetic enough to produce it, hence all events in our model come from DIF flux

$$\nu_\mu\,\mathrm{CH}_2 \;\to\; n\,N_2\,X \;\to\; n\,N_1\,h'\,X \;\to\; N_1\,\gamma\,e^+e^-\,X$$



We note that KARMEN had a energy peaked around 30 MeV, hence the process in our model cannot take place, leading to a null signal prediction.

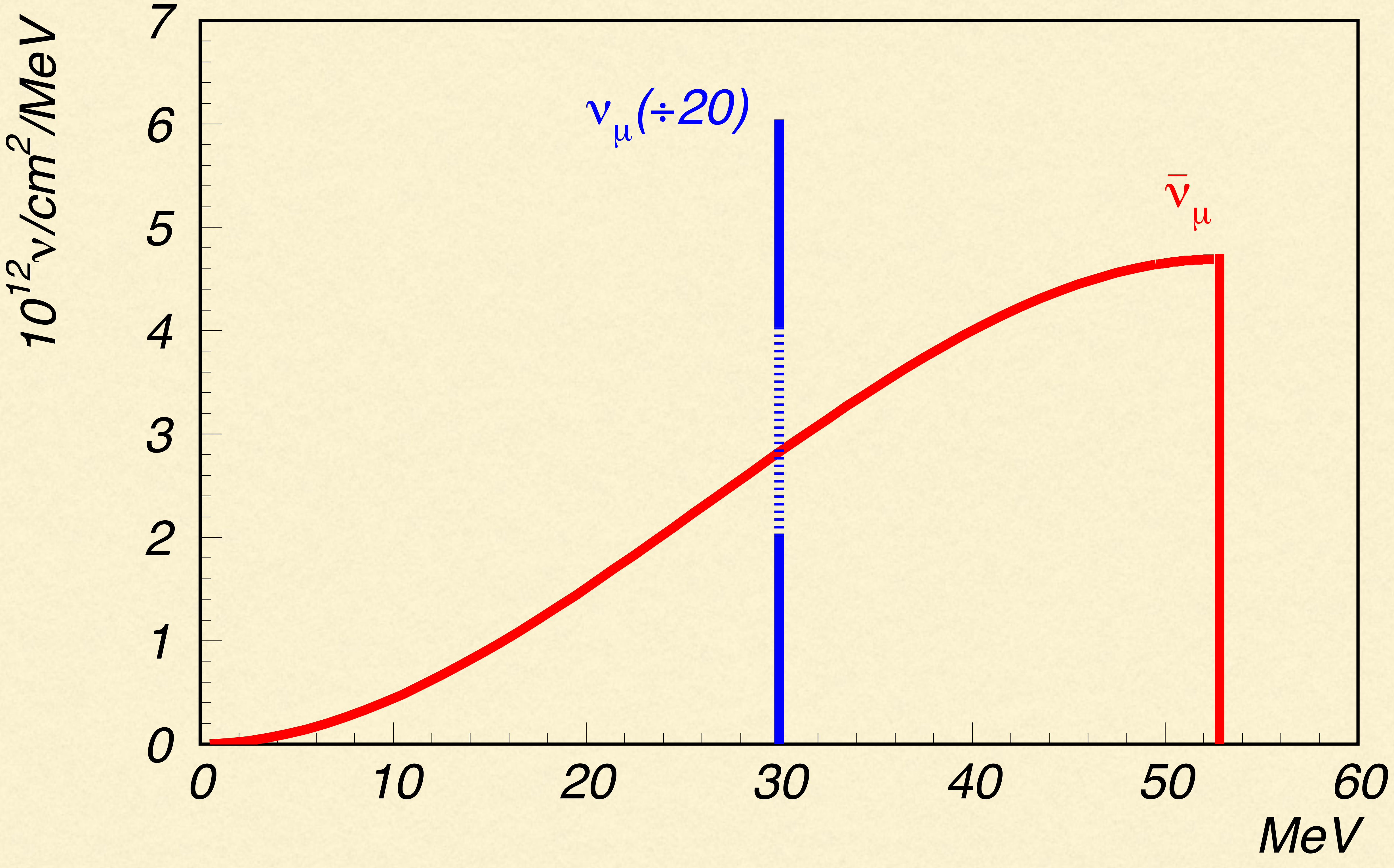


Fig. 3 shows the neutrino energy spectra from the largest DAR sources. The $\bar{\nu}_\mu$ flux from μ^+ DAR provides the neutrinos for the $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation analysis. The ν_e flux from

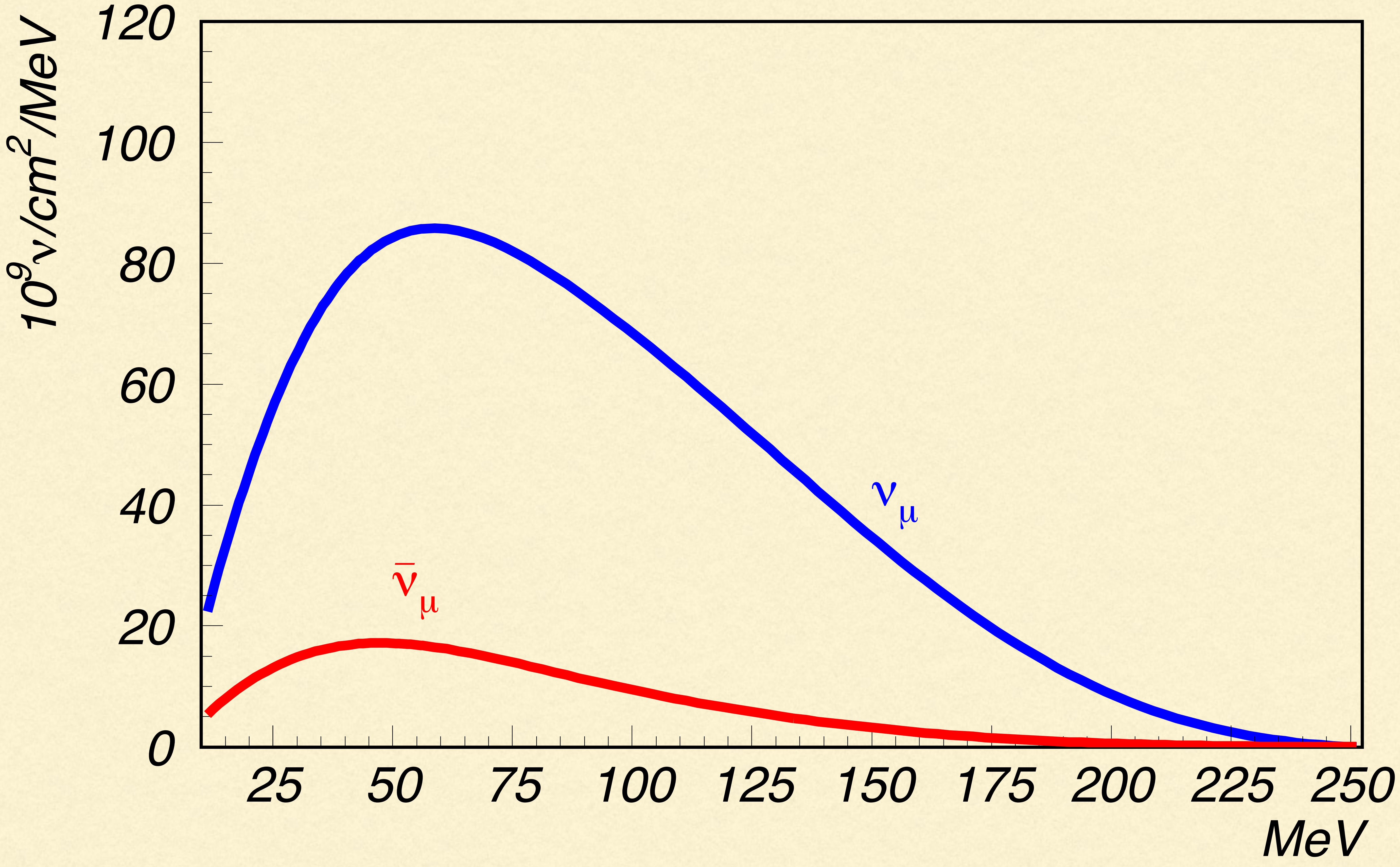


Fig. 4 shows the neutrino energy spectra from various DIF sources averaged over the etector. The ν_μ flux from π^+ DIF provides neutrinos for the $\nu_\mu \to \nu_e$ oscillation analysis.

The ATOMKI anomaly….

Seen in the decay of excited states of ^{8}Be, ^{4}He and recently in ^{12}C

- The emission of a virtual photon by the nucleus, which decays to an e^+e^- pair, (Internal Pair Creation (IPC)), i.e.,

$$p + A \to N^* \to N + e^+ + e^- \,. \tag{5.1}$$

The experiment observes unexpected bumps in the invariant mass and angular separation of the pair, as opposed to SM expectation that both the invariant mass and angular distribution would fall monotonically.

Data is consistent with the production of an new particle X with

$$M_X = 16.7 \pm 0.35(\mathrm{stat}) \pm 0.5(\mathrm{sys}) \quad \mathrm{MeV},$$

From parity and angular momentum conservation, X can be a vector, axial vector or pseudo scalar

The BR fraction is

$$\frac{\mathrm{BR}({}^8\mathrm{Be}^* \to {}^8\mathrm{Be}\,X) \times \mathrm{BR}(X \to e^+e^-)}{\mathrm{BR}({}^8\mathrm{Be}^* \to {}^8\mathrm{Be}\,\gamma)} = 5.8 \times 10^{-6}\,.$$

The observations correspond to an excess of 6.8 sigma

The effective average coupling to nucleons from which one gets couplings to the quarks is, is

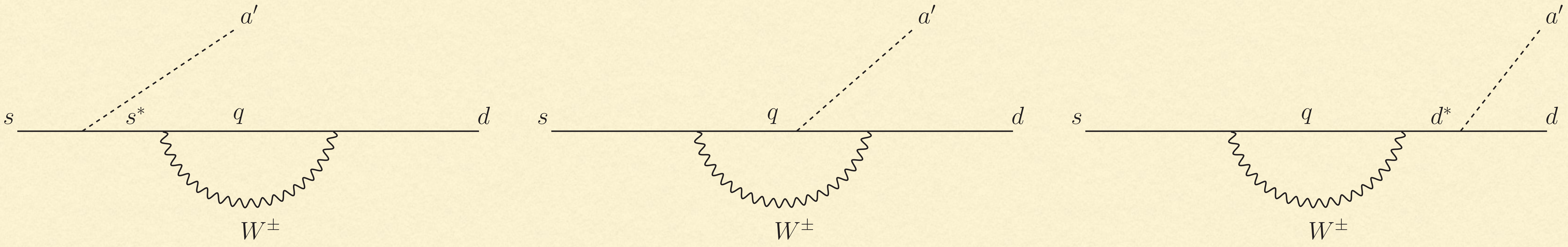
$$\bar{h}_N^2 \equiv \frac{(F_p + F_n)^2}{4}\,.$$

m_{N_1}	m_{N_2}	m_{N_3}	$y_u^{a'}\times 10^5$	$y_e^{a'}\times 10^5$	$y_\mu^{a'}\times 10^5$	$M_{H^\pm}$	$y_c^{a'}\times 10^3$	$y_t^{a'}\times 10^5$
70 MeV	120 MeV	10 GeV	-5.043	2.3	1	305 GeV	6.366	-1.3

$M_{a'}$	M_H	$\sin\xi$	$y_d^{a'}\times 10^5$	$y_{\nu_\mu N_2}^{a'}\times 10^4$	$\lambda_{N_{12}}^{a'}$	M_A	$y_s^{a'}\times 10^5$	$y_b^{a'}$
17 MeV	300 GeV	0.01	-1.3	2.84	0.1	400 GeV	-1.3	0

Couplings to quarks are significantly higher than what they were for MB/LSND alone, in order to obtain a fit identical to the one for MB and LSND alone.

This requires a more careful treatment of constraints , specifically flavour violating meson decays e.g.



Other important constraints come from beam dump experiments, electroweak precision experiments, vacuum stability , unitarity.

Abdallah, RG, Roy, 2010.06159 ;
W. Abdallah, RG, T. Ghosh, N. Khan,
Samiran Roy, Subhojit Roy , 2406.07643

Conclusions……

- Short baseline anomalies like the Ga source anomaly, the RAA, LSND and MB have reached a stage where a host of complementary experiments and theoretical inputs have helped gradually clarify the situation.
  - Improved data on beta spectra and consequent improved flux calculations point to a disappearance of the RAA.

- The situation with the Ga anomaly is unclear, given that the most recent experiment, BEST, verified the presence of the deficit but could not detect any L variation, which would have signalled active sterile oscillations

- Attempts to understand the anomalies using oscillations with eV scale neutrinos show a very strong tension between appearance and disappearance data and with cosmology, while also exhibiting a lack of inner consistency.

The MB and LSND anomalies persist with a high combined statistical significance of 6.1 sigma

Conclusions……

MicroBooNE has recently made important strides in helping establish that SM backgrounds are unlikely to be responsible for the MB signal, strengthening the case that MB and possibly LSND could be signals for new physics.

It is significant that most new physics proposals invoke heavier neutrinos (HNLs)

We have provided an example of such new physics with a light 17 MeV pseudo scalar mediator combined with a second Higgs doublet and 3 RH neutrinos.

The model provides an excellent fit to MB and LSND alone, and to MB, LSND and ATOMKI, and gives SM neutrino mass squared differences in conformity with global oscillation data.

Confirmation of the ATOMKI anomaly by other independent experiments (MEG II, PADME) is important.

A definitive resolution must await results from the Fermilab Short Baseline Program, with its 3 detectors , MicroBooNE, ICARUS and SBND which will test proposals such as ours.

Back-up Slides

Brief Status review of the “other” Short Baseline Anomalies

Neutrino 4 Experiment: Results/Criticisms

Neutrino-4 is a reactor neutrino experiment designed to search for short-baseline sterile neutrino oscillations motivated primarily by the Reactor Antineutrino Anomaly

Liquid scintillator with 0.1% gadolinium concentration for detection of the neutron in the ¯ve + p → n + e + detection process for the reactor antinus

Claims Sterile-active oscillations at CL 3.5σ in the vicinity of Δm^2_14 ≈ 7.26eV^2 and sin^2 2θ_14 ≈ 0.38 .

arXiv:1809.10561 [hep-ex]

Criticism that the Neutrino-4 results can only be reproduced when neglecting the detector energy resolution.

arXiv:2101.06785. Giunti et al

In conflict with exclusion curves of other reactor experiments ; PROSPECT, STEREO, DANSS, RENO NEOS

Short Baseline Anomalies and Sterile Neutrinos

In the absence of any new physics signals at the Large Hadron Collider (LHC), anomalous results at low energy experiments have become the subject of increased attention and scrutiny.

Over the past couple of decades, a number of anomalous results have been observed in experiments which involve the production and detection of neutrinos over short baselines (< 1 km).

Sterile neutrinos of $(\text{mass})^2 = \text{eV}^2$ and consequent active-sterile oscillations have been invoked to explain them.

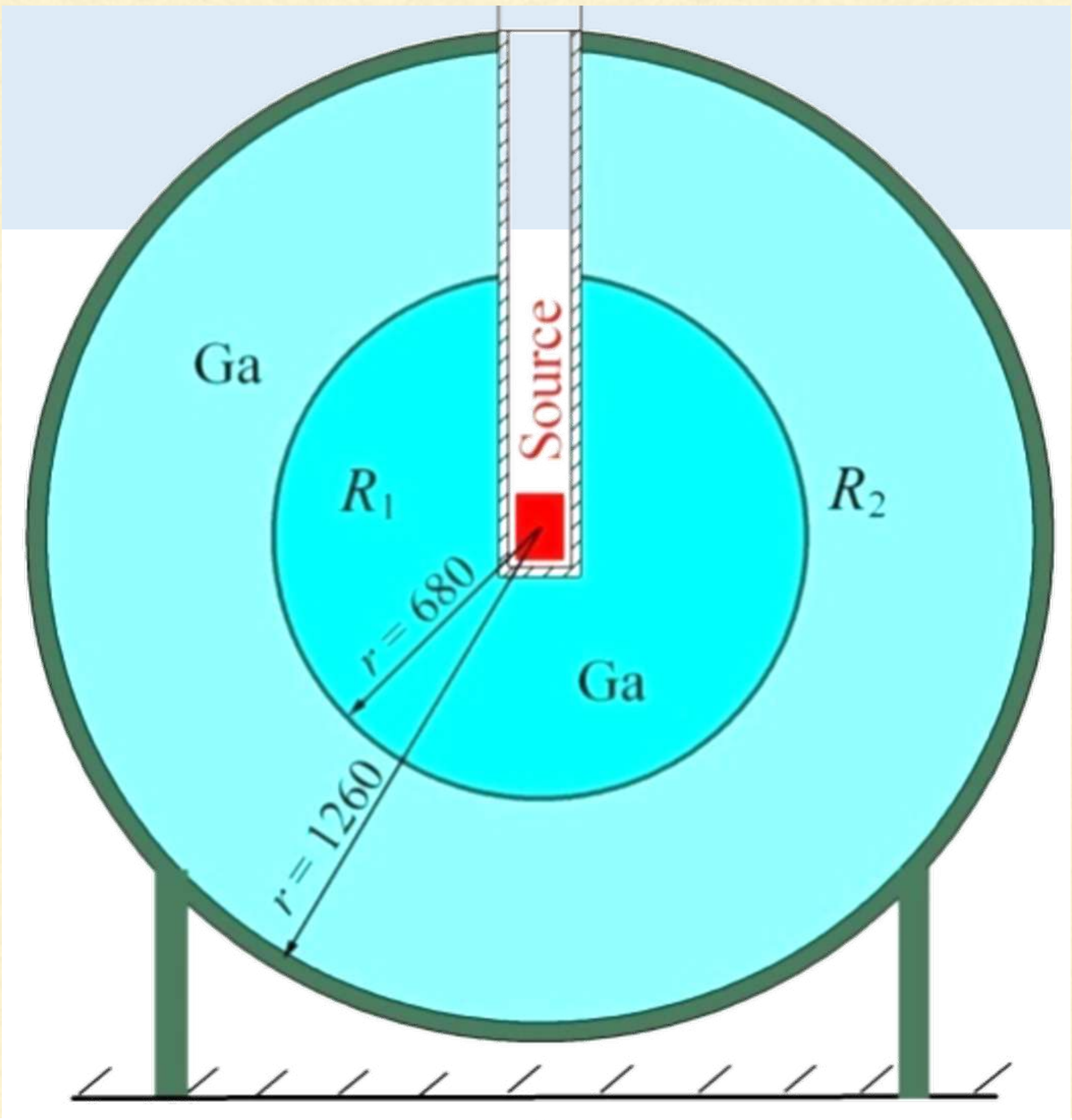
This hypothesis has come under increasing pressure from recent experimental data (IceCube, MicroBooNE), joint oscillation analyses, cosmology and the requirement of mutual consistency.

Is other new physics responsible for these anomalies?

The Gallium source Anomaly:

Intense radioactive sources (e.g. Cr, Ar) with well-determined neutrino spectra are used. These neutrinos are captured by Ga via $\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$

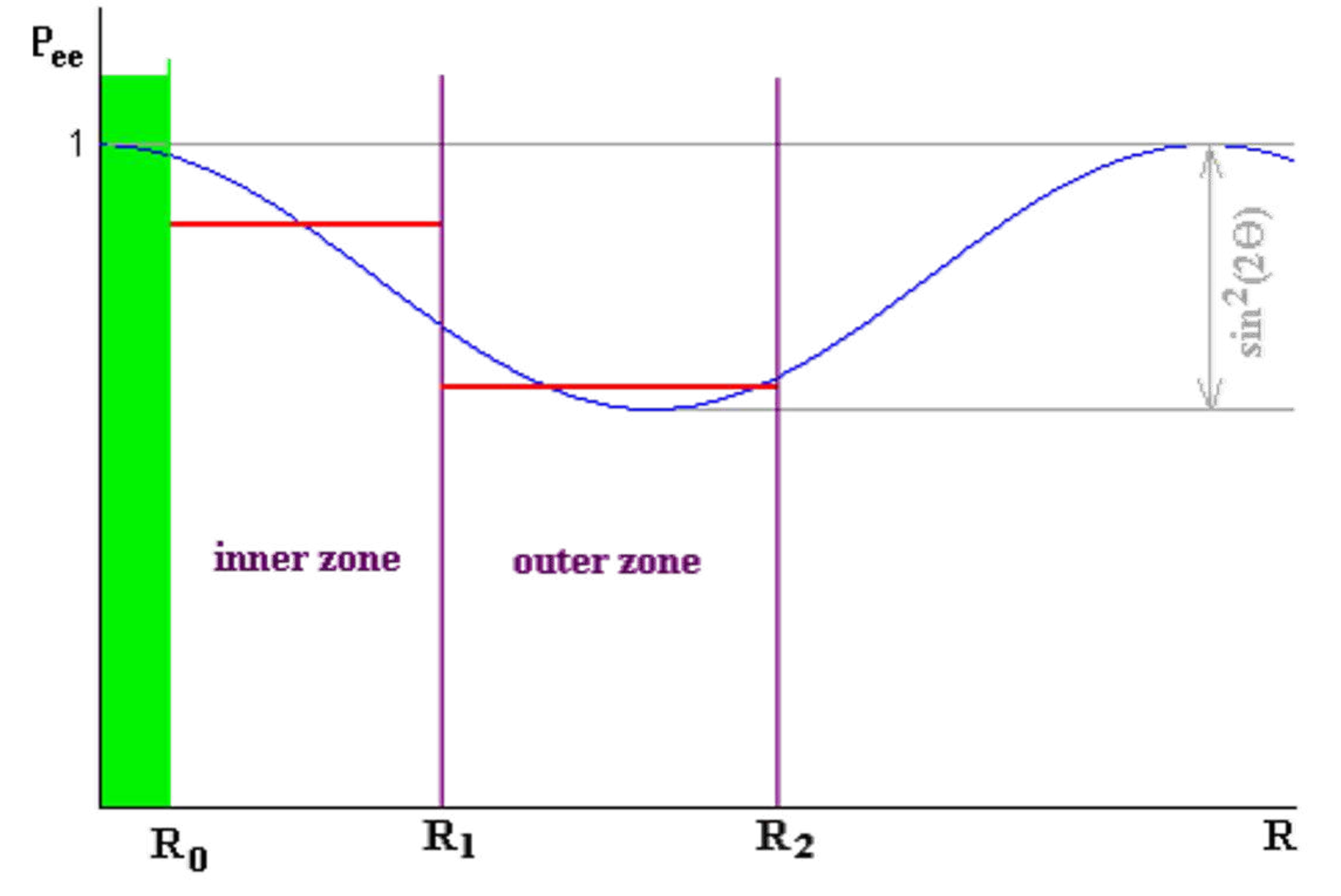
Baselines over which the decay neutrinos propagate are very short, ~ 1 m. However, in the latest experiment (BEST) 2 target zones are created, to see evidence of oscillations.

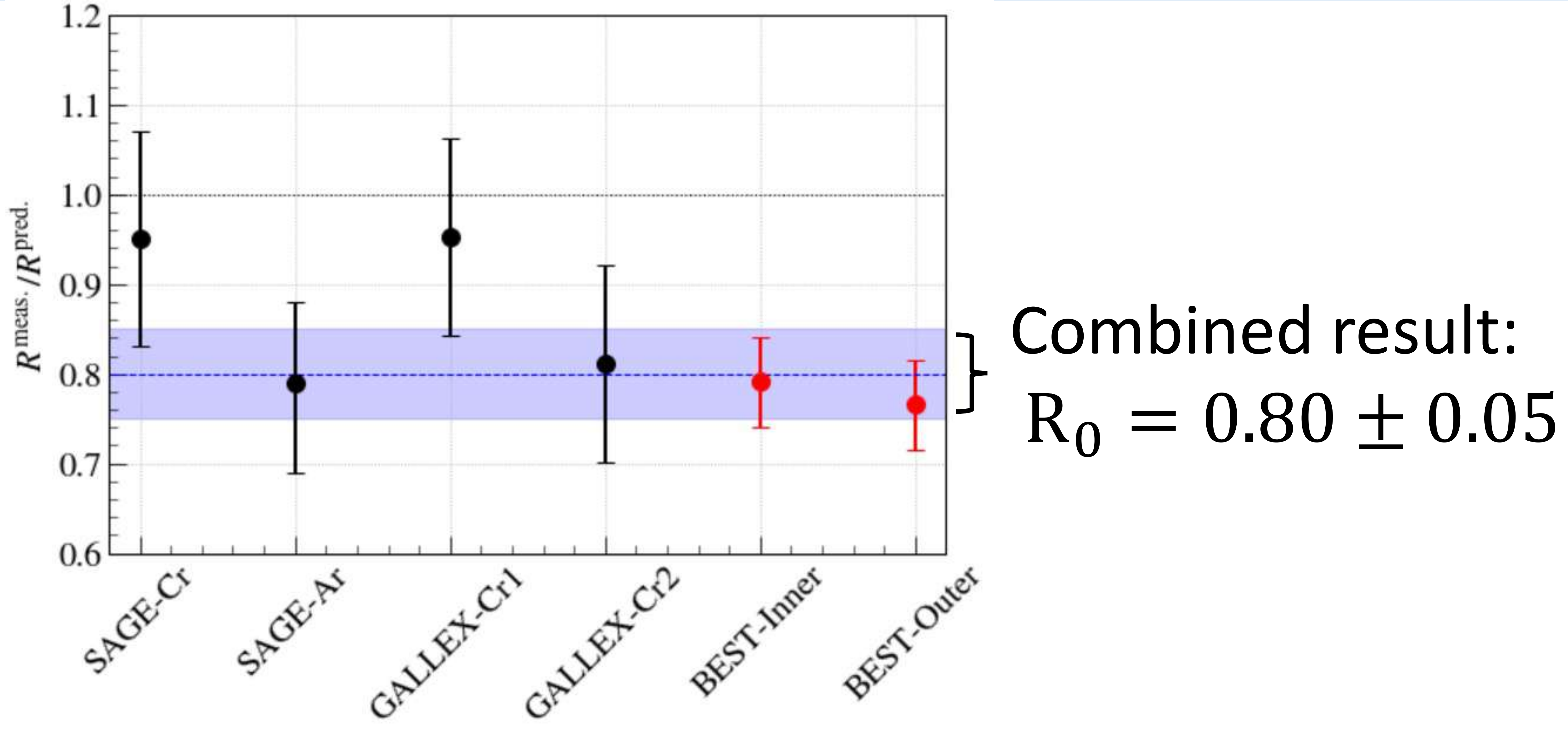


- Radio chemistry for extraction and counting of the ^{71}Ge was developed in SAGE solar measurements. and is well understood

Anomalies at Short Baselines…….1) The Gallium source Anomaly

If one were to understand the SAGE and Gallex results in terms of sterile neutrino oscillations, one would expect these results (shown adjacent) in BEST





Combined result:

$R_0 = 0.80 \pm 0.05$

BEST confirms (with higher statistical precision) (4σ) a deficit in overall flux consistent with earlier SAGE/GALLEX results.

Barinov et al, arXiv:2109.11482, PRL arXiv:2201.07364, PRC 105, (2022) no. 6, 060552

Anomalies at Short Baselines…….1) The Gallium source Anomaly

However, while results can be accommodated in the sterile/active oscillation space, BEST did not observe any variation with distance.

Note that large mixing is required for the oscillation interpretation.

Smoking gun for oscillations is missing

This conflicts with:

——Reactor $\bar{\nu}_e$ which requires much smaller θ_{ee}

(slides below) →

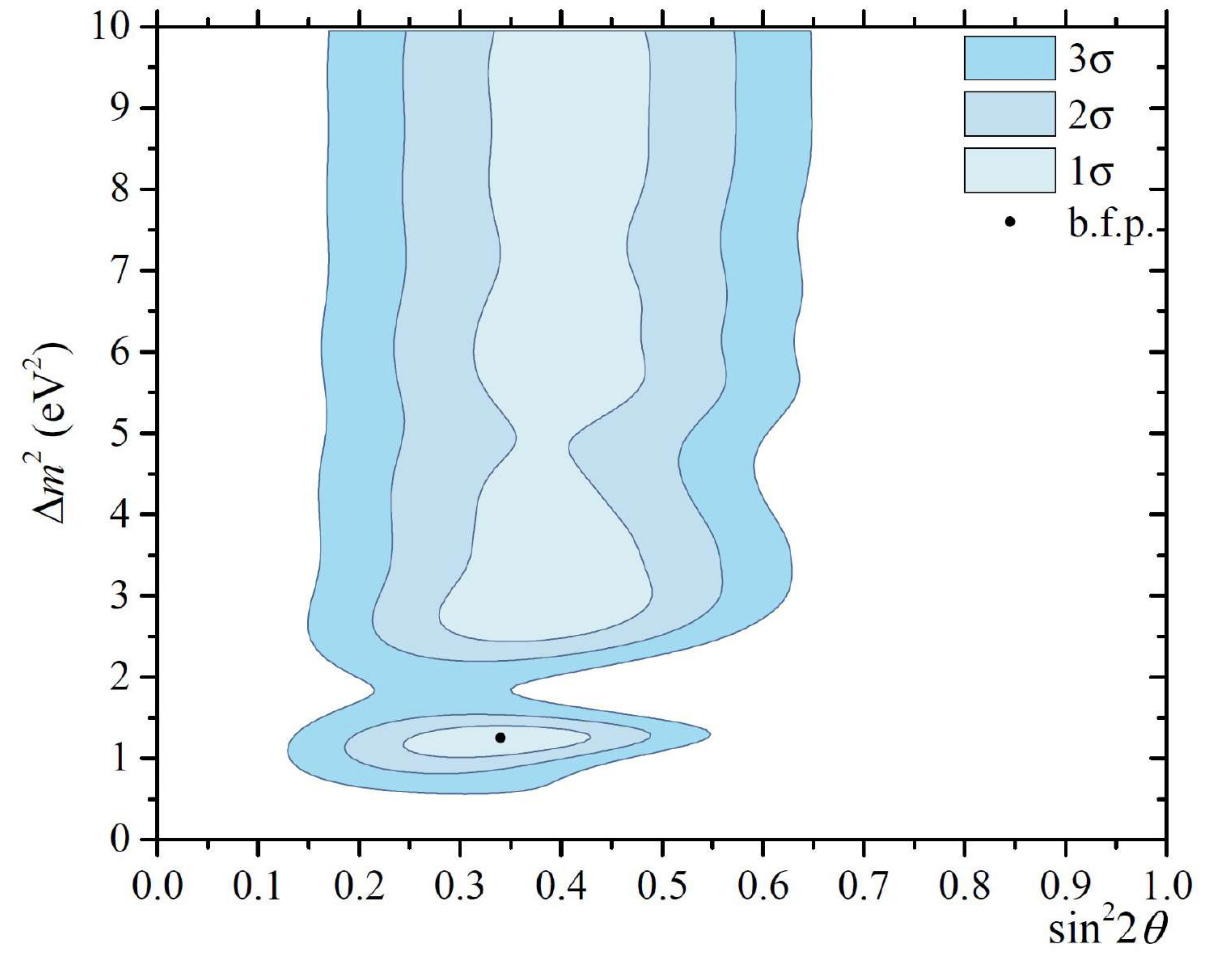


FIG. 8. Allowed regions for two GALLEX, two SAGE and two BEST results. The best-fit point is $\sin^2 2\theta$=0.33, Δm^2 = 1.25 eV^2 and is indicated by a point.

——Solar data, which do not tolerate high θ_{ee}

Possible non-oscillation reasons for the observed deficit could be inaccuracies in 1) xsecs, 2) source strength, 3) counting efficiency 4) extraction efficiency.

No clear answer at present.

Anomalies at Short Baselines…….Reactor Antineutrino Anomaly (RAA)

Reactor antineutrinos are produced from beta decays of neutron-rich fission fragments generated by the heavy isotopes 235U, 238U, 239Pu, and 241Pu

The most important antineutrino fluxes are those produced by the fissions of 235U and 239Pu.

The flux measurement from various reactors, was, until recently, on the average, about 3.5% (~3σ) lower than predicted from careful calculations done by several groups.

Mueller et al. 1101.2663, Huber 1106.0687, Giunti et al. 2110.06820

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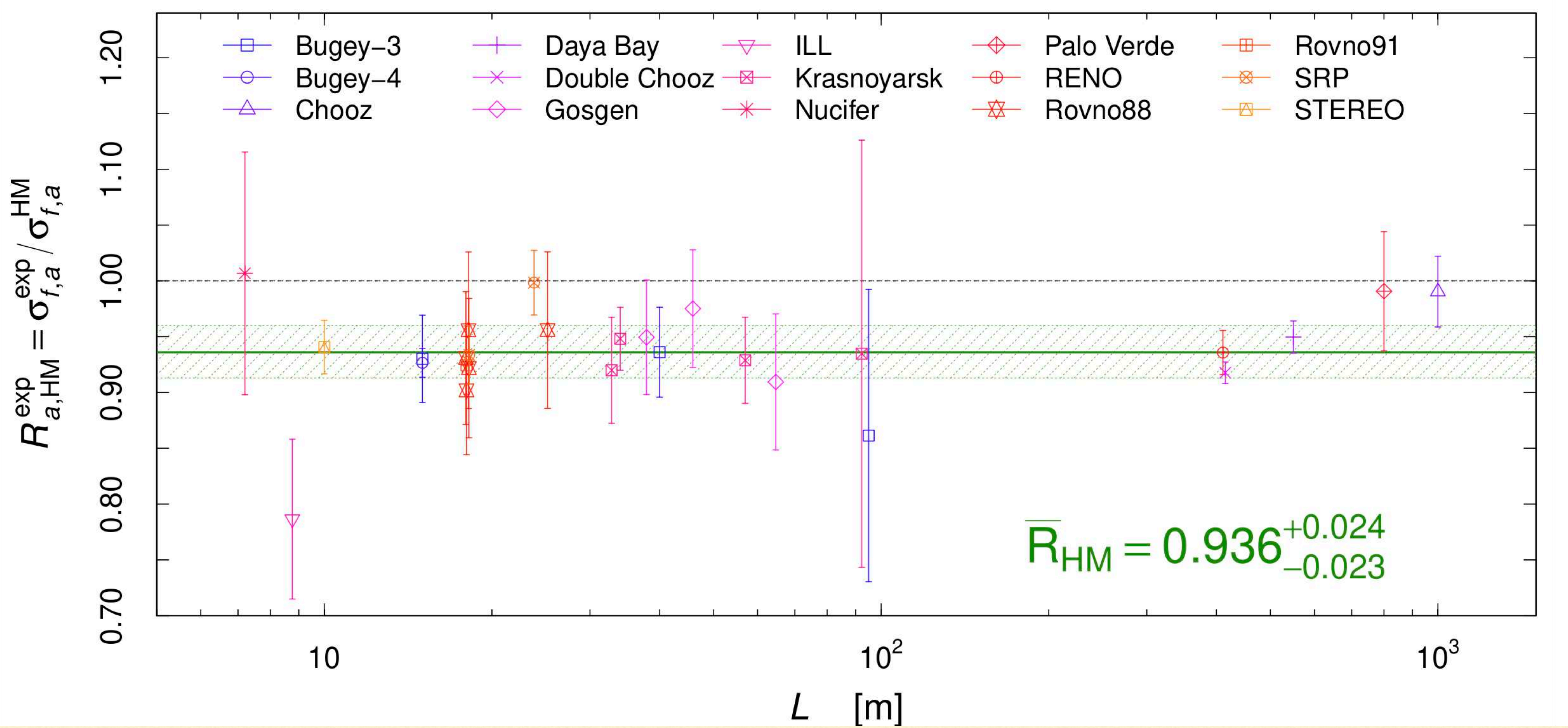
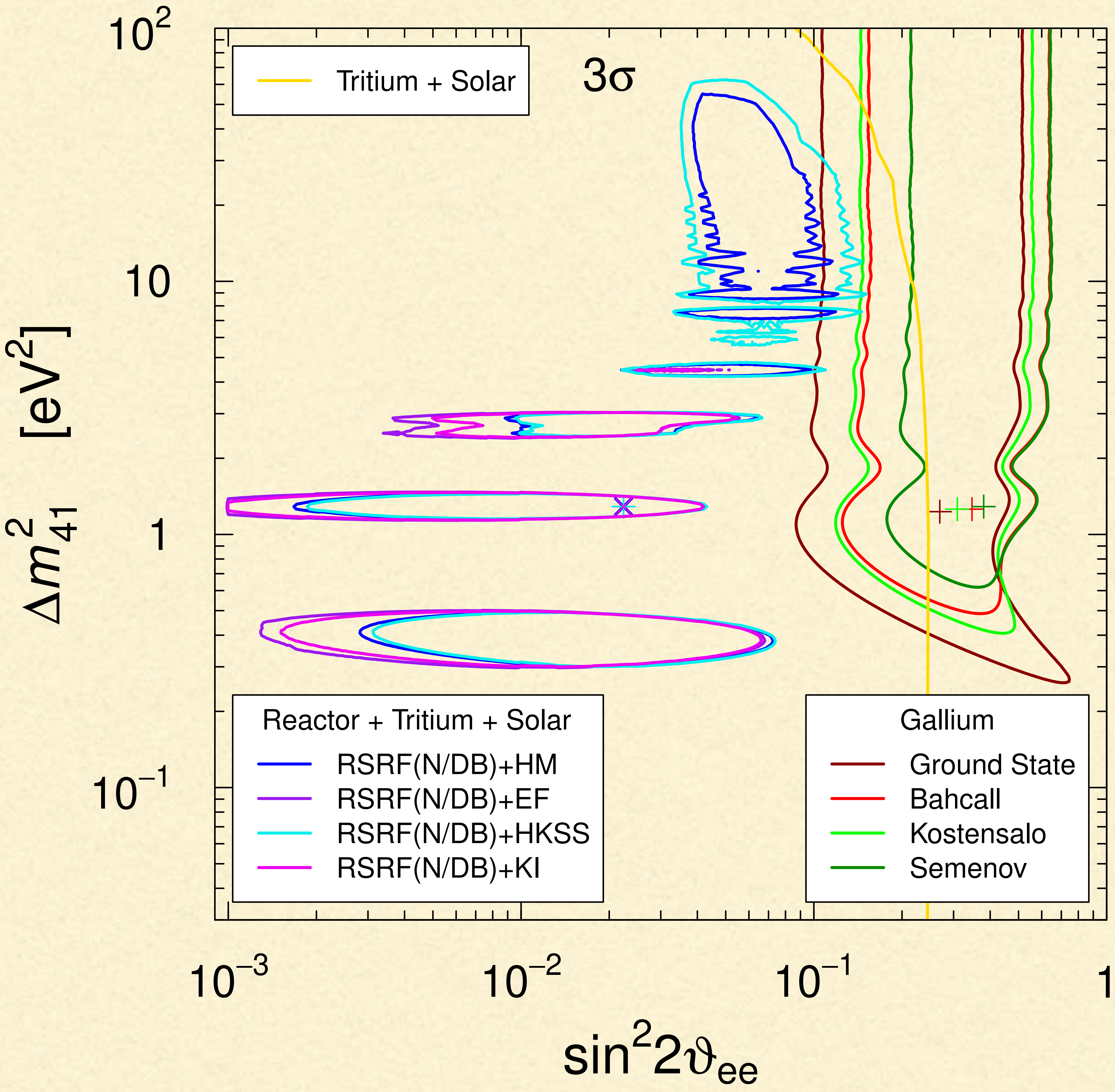
Anomalies at Short Baselines…….Reactor Antineutrino Anomaly (RAA)



Mueller et al. 1101.2663, Huber 1106.0687, Giunti et al. 2110.06820

This raised the possibility that the deficit was due to active-sterile oscillations .



Allowed oscillation regions for RAA in strong tension with oscillation parameters required to explain the Ga anomaly.

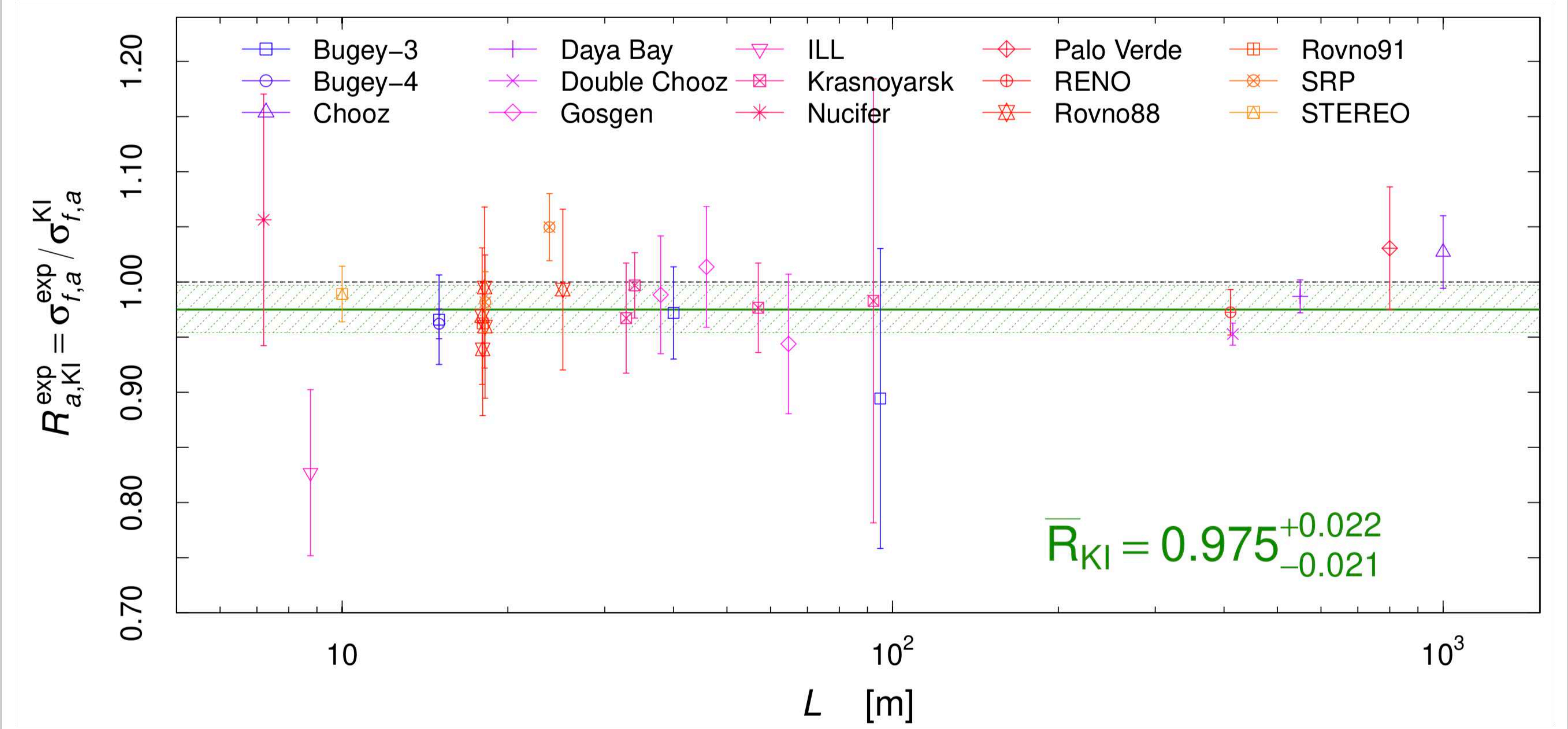
Anomalies at Short Baselines…….Reactor Antineutrino Anomaly (RAA)

Nuclear databases have been improved in recent years, especially through the application of the Total Absorption Gamma-ray Spectroscopy (TAGS) technique for a better identification of the β decay branches.

This new information was used by Fallot et al [18] (EF model) (1904.09358), and Silaeva et al, 2012.09917 to obtain a 235U reactor antineutrino flux that is smaller than that of the earlier models.

This has led to improved agreement with measured fluxes, and there is now a belief in the community that the RAA has been understood to be a flux calculation/data issue (as opposed to a neutrino deficit issue) .

Anomalies at Short Baselines…….Reactor Antineutrino Anomaly (RAA)

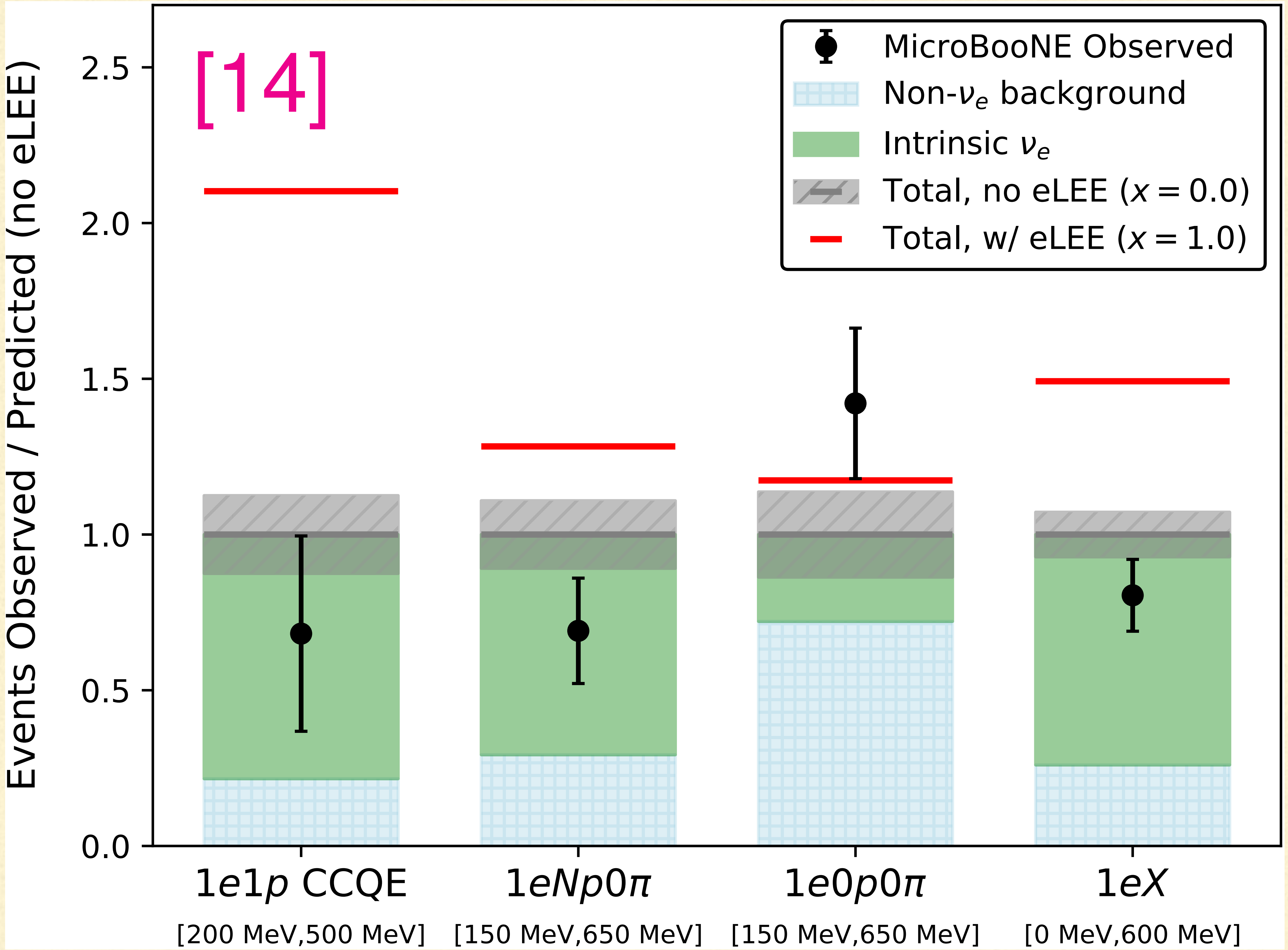


Kopeikin et al 2103.01684, Berryman et al 2005.01756, Giunti et al 2110.06820

MicroBooNE ……….

MicroBooNE has found no evidence for any additional π^0 or γ production which may simulate an electron-like signal in MB.

A search for νe induced interactions has also not provided any evidence of an excess.

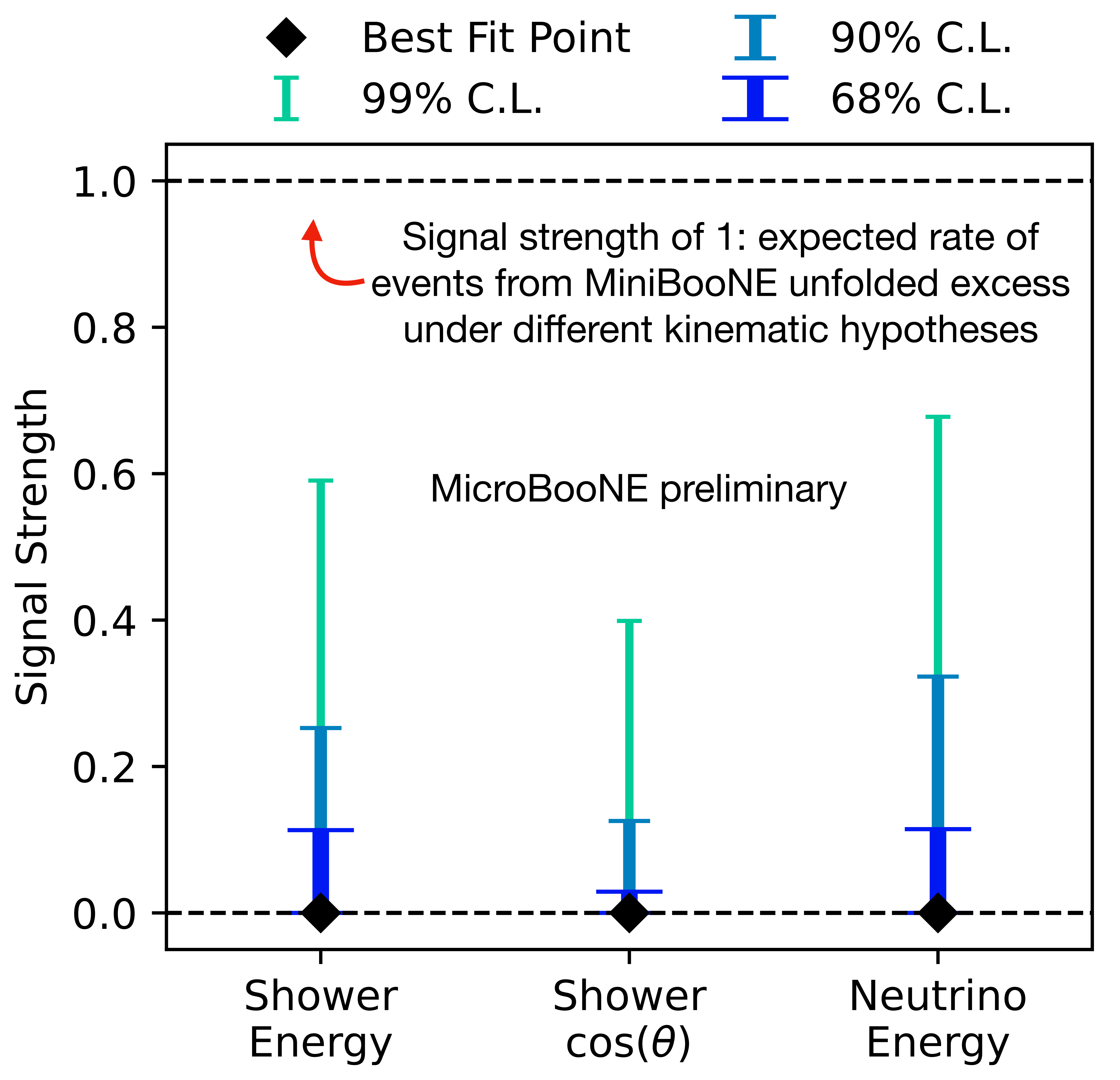


Maltoni, Nu 2024 talk

MicroBooNE ……….latest results

- data compatible with background-only prediction
- **data inconsistent with ν_e-like excess at > 99% CL**
- results consistent across kinematic variables tested.

Caratelli, (MicroBooNE collab) Nu 2024 talk



LSND useful……

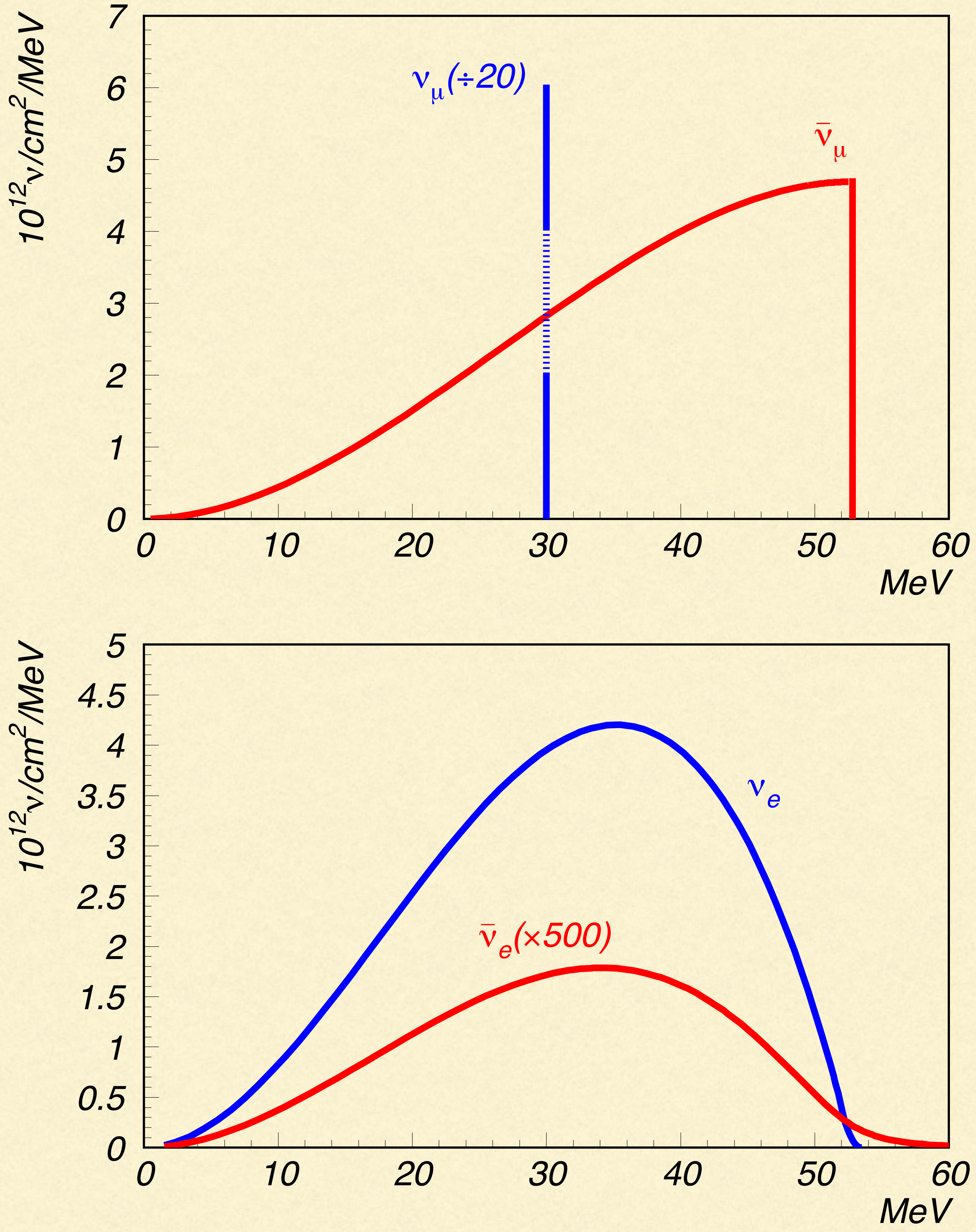


FIG. 3: The decay-at-rest neutrino fluxes averaged over the detector.

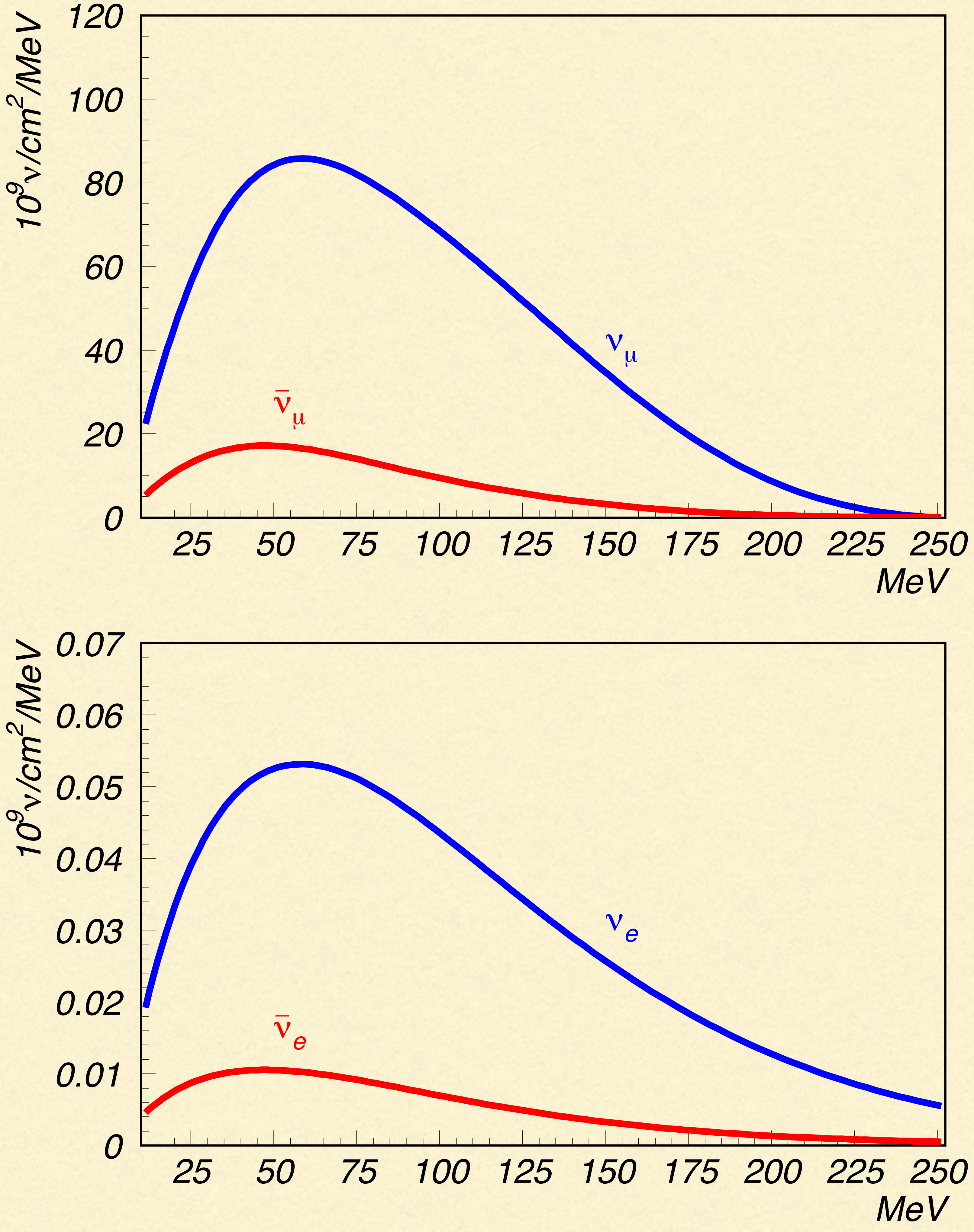


FIG. 4: The decay-in-flight neutrino fluxes averaged over the detector.

Short Baseline Neutrino Program at Fermilab

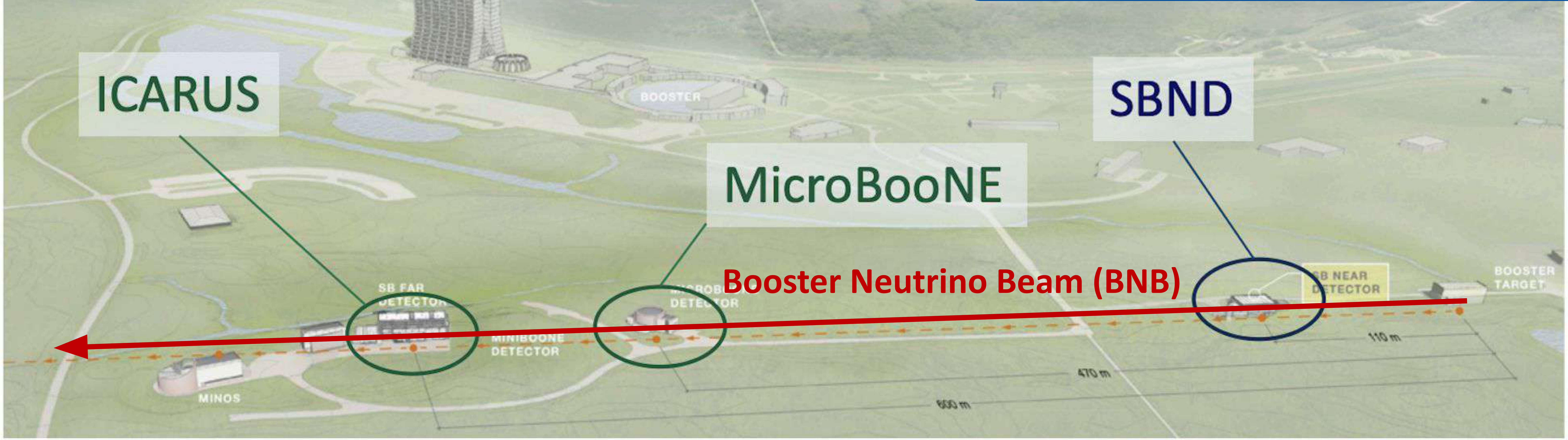
Anne Schukraft talk at Neutrino 2022

ICARUS

600m baseline
470t active volume
Commissioning

SBND

110m baseline
112t active volume
Under Construction

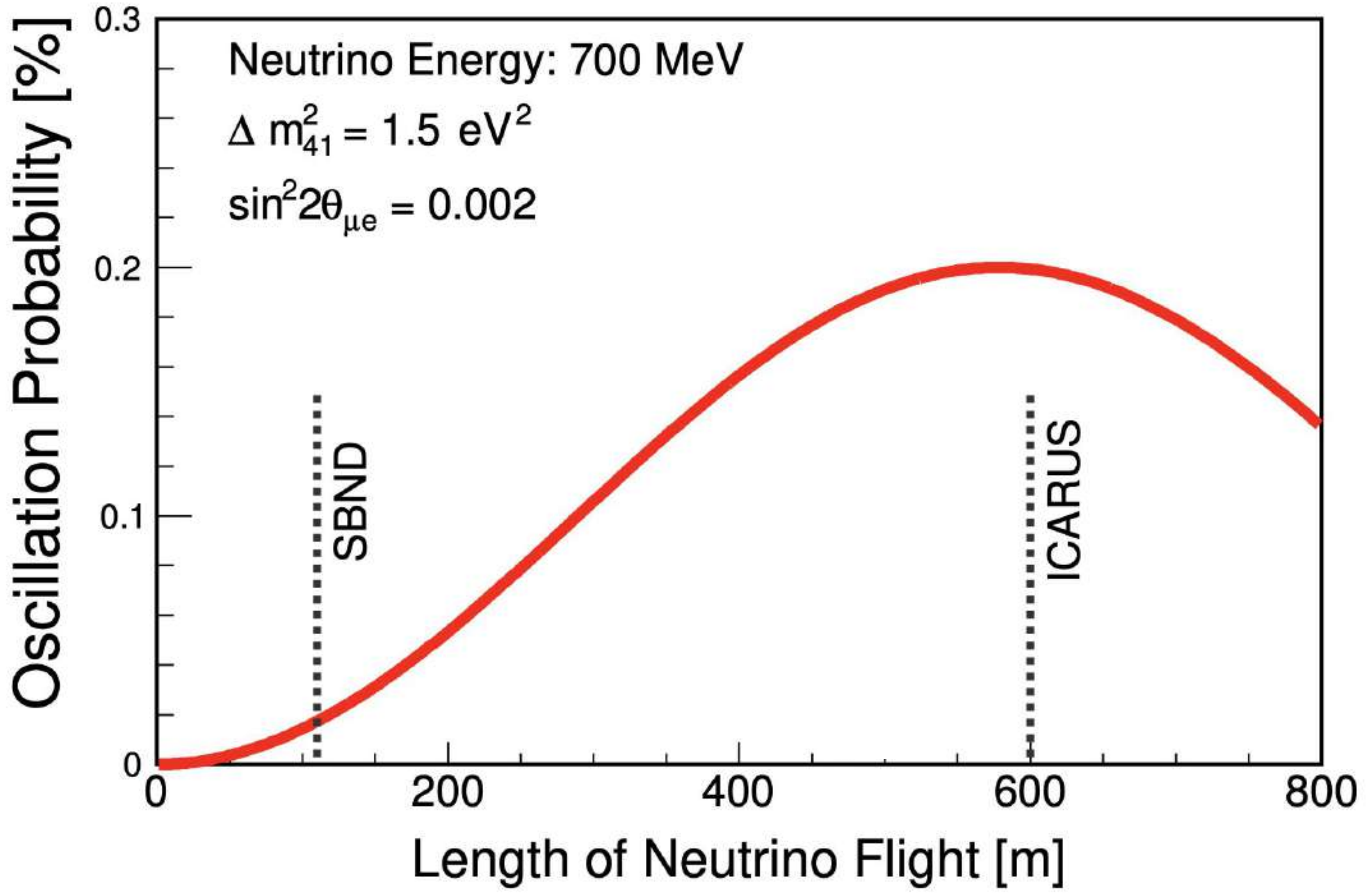


- Three detectors sampling the *same neutrino beam* at different distances

SBN Oscillation Sensitivity

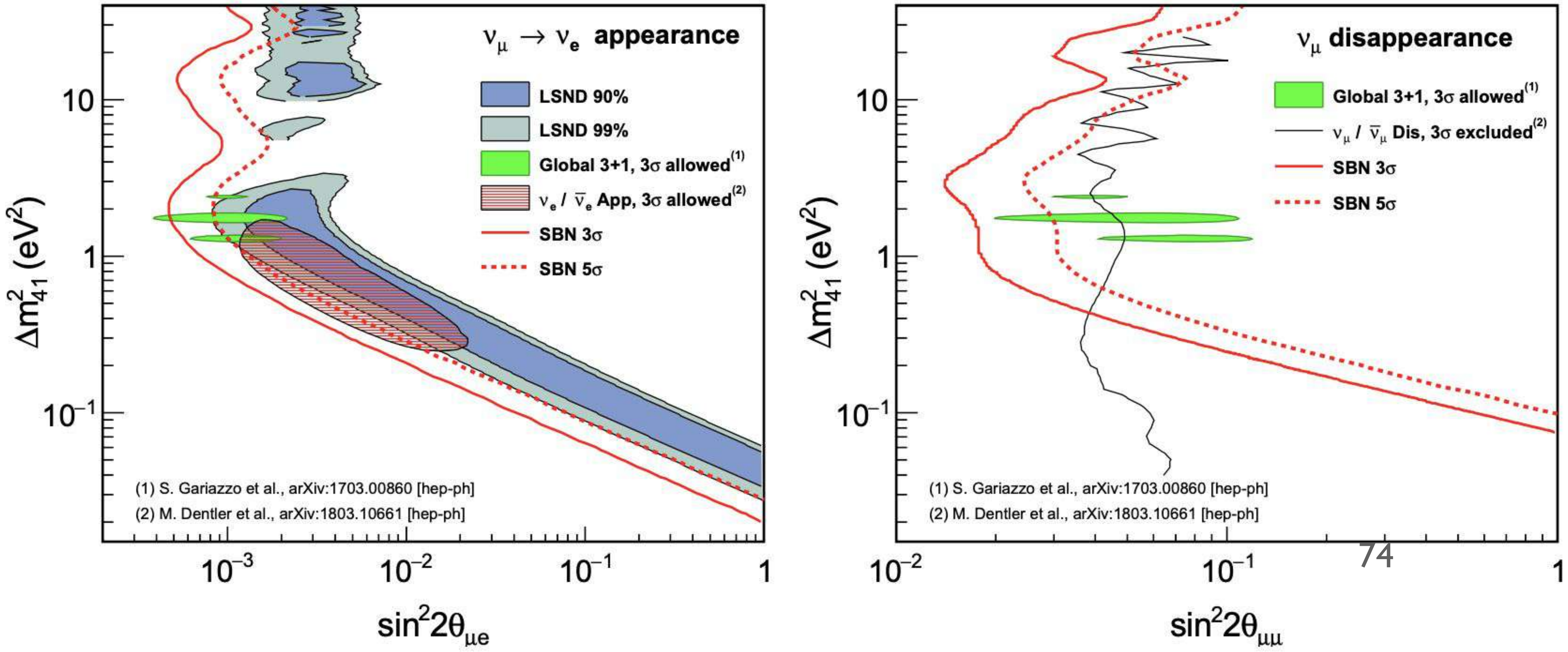
Anne Schukraft talk at Neutrino 2022

- SBND + ICARUS will test the sterile neutrino hypothesis
  - can cover the parameter space favored by past anomalies with 5σ significance
- Observing neutrino flux at different distances from the beam target
- Effective systematics constraint through near detector (SBND) and same detector technology in near and far detector
- **Search for appearance of ν_e and disappearance of ν_μ within the same experiment**
  - current results show a 4.7σ tension between ν_e appearance and ν_μ disappearance channels



P. Machado et al, arXiv:1903.04608V11

(SBN sensitivities for 6.6 x 10^{20} protons on the BNB target)



Standard Neutrino oscillations…….in the vacuum

$$P(\nu_e \to \nu_\mu; L) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right),$$

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) = 1 - P(\nu_\mu \to \nu_e) = P(\nu_\mu \to \nu_\mu),$$

$$P(\nu_\alpha \to \nu_\beta; L) = \delta_{\alpha\beta} - 4\sum_{i>j}\mathrm{Re}(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right)$$
$$+\,2\sum_{i>j}\mathrm{Im}(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right),$$

$\alpha, \beta = e, \mu, \tau$.

Standard Neutrino oscillations…….in the vacuum

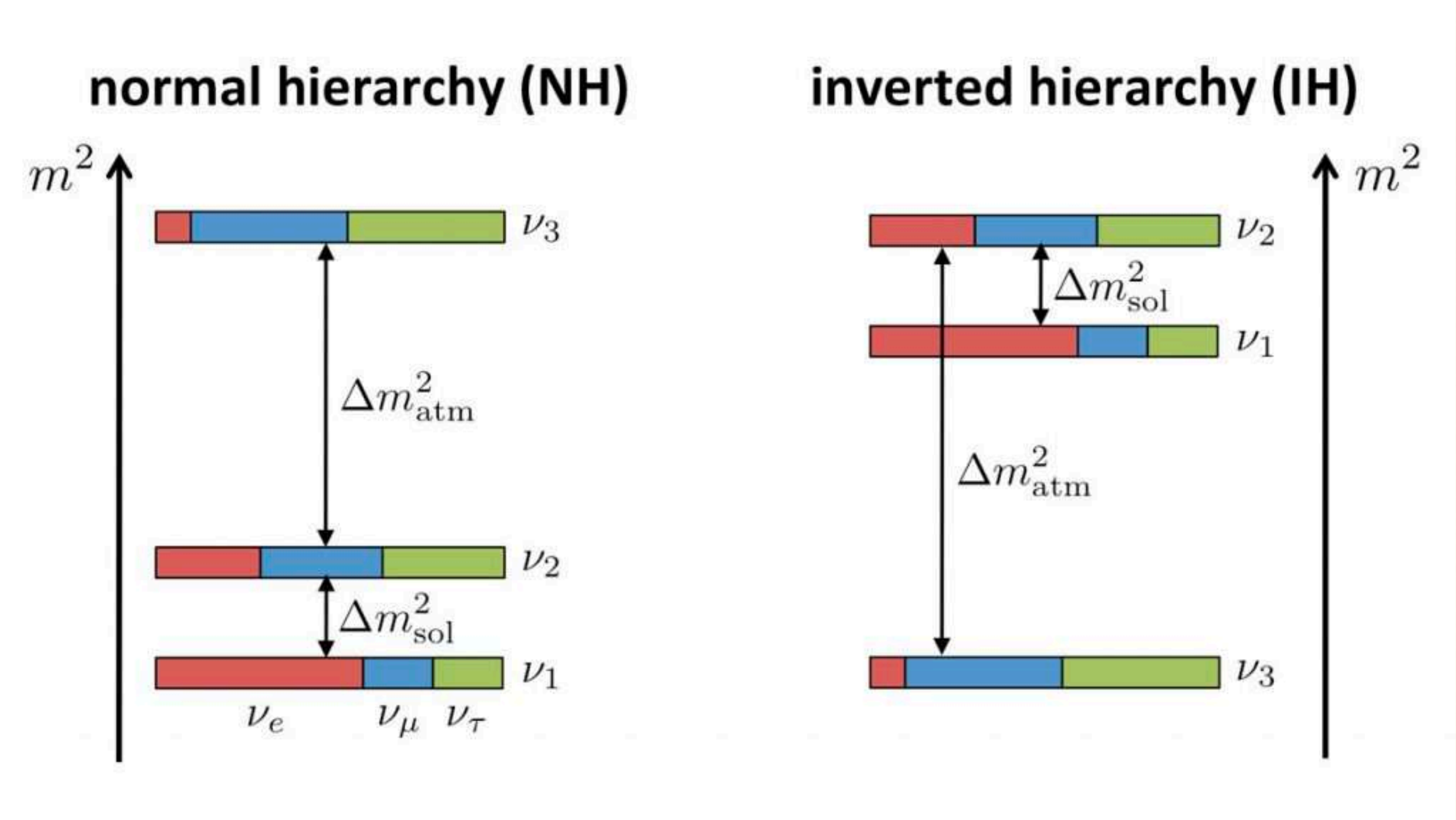
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} ,$$

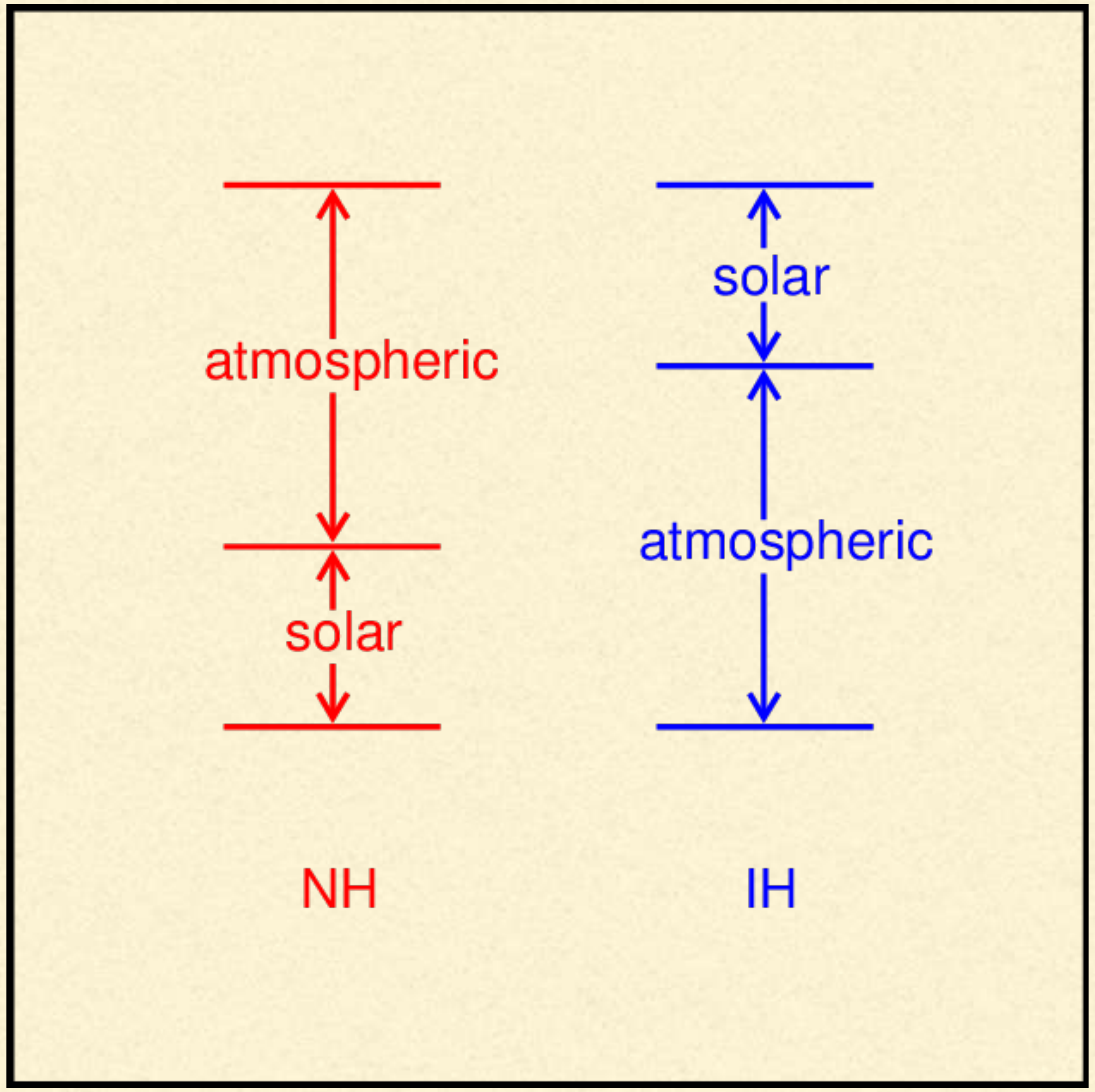
U relates the weak interaction eigenstates and the mass eigenstates through the leptonic mixing parameters θ_{12}, θ_{13}, θ_{23}, δ (the Dirac CP-violating phase), as well as ρ and σ (the Majorana CP-violating phases).

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{ij} \equiv \cos(\theta_{ij})$ and $s_{ij} \equiv \sin(\theta_{ij})$.

Mass hierarchy of neutrinos





Useful SBL formulae

$$P_{\alpha\beta} = \sum_{j,k=1}^{4} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \exp\left[-i \frac{\Delta m^2_{jk} L}{2E} \right].$$

General, for all baselines

$$U \equiv R_{34}(\theta_{34})\, R_{24}(\theta_{24}, \delta_{24})\, R_{14}(\theta_{14})\, R_{23}(\theta_{23})\, R_{13}(\theta_{13}, \delta_{13})\, R_{12}(\theta_{12}, \delta_{12})\,, \qquad (2)$$

where $R_{ij}(\theta_{ij})$ denotes a real rotation matrix in the (ij)-plane with rotation angle θ_{ij}, and $R_{ij}(\theta_{ij}, \delta_{ij})$ includes in addition a complex phase δ_{ij}. In most cases, however, we will present

For the following discussion the so-called short-baseline limit of eq. (1) will be useful. This limit refers to the situation where $\Delta m^2_{21} L/4E \ll 1$, $\Delta m^2_{31} L/4E \ll 1$, so that standard three-flavor oscillations have not had time to develop yet. In this case, eq. (1) generically simplifies to

$$P^{\rm SBL}_{\alpha\alpha} = 1 - 4|U_{\alpha 4}|^2 (1 - |U_{\alpha 4}|^2)\, \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right), \qquad (3)$$

$$P^{\rm SBL}_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2\, \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right). \qquad (\alpha \neq \beta) \qquad (4)$$

$$\sin^2 2\theta_{\mu e} \equiv 4|U_{e4}|^2 |U_{\mu 4}|^2\,.$$

Useful SBL formulae

The high-energy IceCube analysis from ref. [52] exploits the fact that active-to-sterile neutrino oscillations in matter are resonantly enhanced by the MSW effect [55, 56] at an energy of

$$E_{\rm res} = 5.3\ \text{TeV} \times \left(\frac{5\ \text{g/cm}^3}{\rho_\oplus}\right)\left(\frac{\Delta m^2_{41}}{1\ \text{eV}^2}\right) . \qquad (8)$$

The effective mixing angles $\theta_{\alpha\beta}$ for short-baseline oscillations are defines below

$$P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha\beta} + (-1)^{\delta_{\alpha\beta}} \cdot \boxed{\sin^2 2\theta_{\alpha\beta}} \cdot \sin^2(1.267 \frac{\Delta m^2_{41} L}{E})$$

ν_e disappearance $\boxed{\sin^2 2\theta_{ee} = \sin^2 2\theta_{14}}$

ν_μ disappearance $\boxed{\sin^2 2\theta_{\mu\mu} = 4\cos^2\theta_{14}\sin^2\theta_{24}\left(1 - \cos^2\theta_{14}\sin^2\theta_{24}\right)}$

ν_e appearance $\boxed{\sin^2 2\theta_{\mu e} = \sin^2 2\theta_{14}\sin^2\theta_{24}}$

- non-zero ν_e appearance requires both ν_e and ν_μ disappearances

$$P_{ee} \simeq 1 - \sin^2 2\vartheta_{ee} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) .$$

Useful SBL formulae. (2210.10216)

$$|U_{e4}|^2 = \sin^2\theta_{14},$$
$$|U_{\mu 4}|^2 = \cos^2\theta_{14}\ \sin^2\theta_{24},$$
$$|U_{s4}|^2 = \cos^2\theta_{14}\ \cos^2\theta_{24}\ \cos^2\theta_{34},$$

$$\Delta_{41} \equiv \frac{\Delta m^2_{41} L}{4E} = 1.267 \left(\frac{\Delta m^2_{41}}{\mathrm{eV}^2}\right)\left(\frac{\mathrm{MeV}}{E}\right)\left(\frac{L}{\mathrm{m}}\right)$$

$$\sin^2 2\theta_{\alpha\beta} = 4|U_{\alpha 4}|^2|\delta_{\alpha\beta} - |U_{\beta 4}|^2|.$$

$$\sin^2 2\theta_{ee} = \sin^2 2\theta_{14},$$
$$\sin^2 2\theta_{\mu e} = \sin^2 2\theta_{14}\ \sin^2\theta_{24},$$
$$\sin^2 2\theta_{\mu\mu} = 4\cos^2\theta_{14}\sin^2\theta_{24}(1 - \cos^2\theta_{14}\sin^2\theta_{24}),$$
$$\sin^2 2\theta_{es} = \sin^2 2\theta_{14}\ \cos^2\theta_{24}\ \cos^2\theta_{34},$$
$$\sin^2 2\theta_{\mu s} = \cos^4\theta_{14}\ \sin^2 2\theta_{24}\ \cos^2\theta_{34}.$$

Notes on excess in 1e0p0pi channel in MicroB

Each selection shows a strong preference for the absence of an electron-like MiniBooNE signal, with the exception of the 1e0p0π se- lection, driven by a data excess in the lowest energy bins, which also contain the highest contributions from non-νe backgrounds.

With the exception of the 1e0p0π selection which is the least sensitive to a simple model of the MiniBooNE low-energy excess, MicroBooNE rejects the hypothesis that νe CC interactions are fully responsible for that ex- cess (x = 1) at >97% CL for both exclusive (1e1p CCQE, 1eNp0π) and inclusive (1eX) event classes.

Useful SBL formulae. (Caratelli talk, MicroB, Nu 2024)

3+1 parametrization

Full 3+1 search ⟶

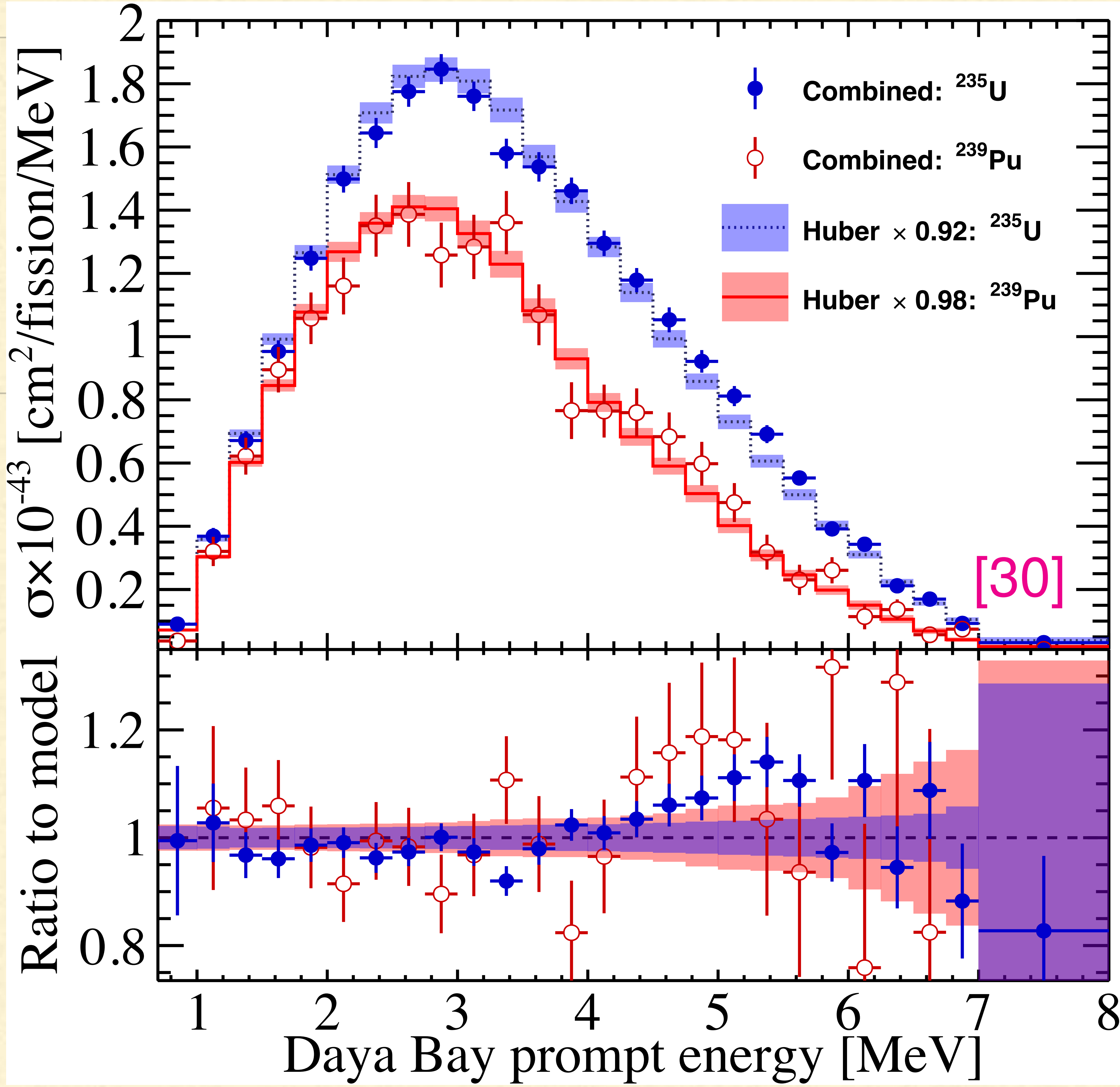
$$P_{\nu_e\to\nu_e} = 1 - 4(1-|U_{e4}|^2)|U_{e4}|^2\sin^2\Delta_{41},$$

$$P_{\nu_\mu\to\nu_\mu} = 1 - 4(1-|U_{\mu4}|^2)|U_{\mu4}|^2\sin^2\Delta_{41},$$

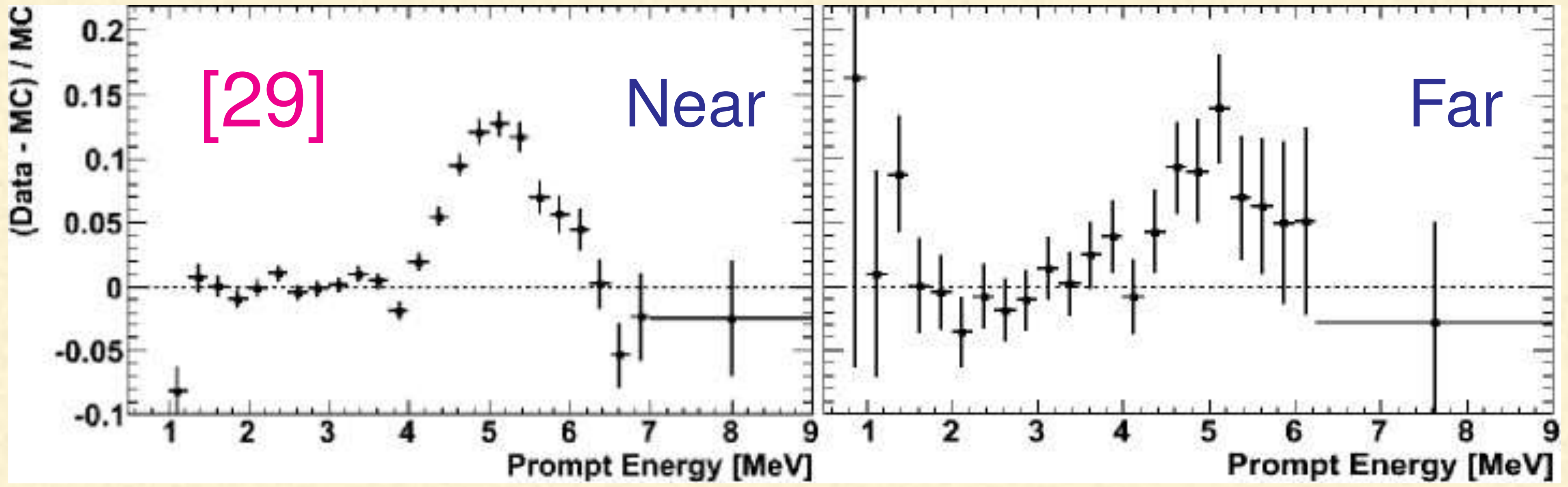
$$P_{\nu_\mu\to\nu_e} = 4|U_{\mu4}|^2|U_{e4}|^2\sin^2\Delta_{41}.$$

$\sin^2 2\theta_{ee}$	$= \sin^2 2\theta_{14}$	$= 4(1-\lvert U_{e4}\rvert^2)\lvert U_{e4}\rvert^2$
$\sin^2 2\theta_{\mu\mu}$	$= 4\cos^2\theta_{14}\sin^2\theta_{24}\left(1-\cos^2\theta_{14}\sin^2\theta_{24}\right)$	$= 4(1-\lvert U_{\mu4}\rvert^2)\lvert U_{\mu4}\rvert^2$
$\sin^2 2\theta_{\mu e}$	$= \sin^2 2\theta_{14}\sin^2\theta_{24}$	$= 4\lvert U_{\mu4}\rvert^2\lvert U_{e4}\rvert^2$
$\sin^2 2\theta_{es}$	$= \sin^2 2\theta_{14}\cos^2\theta_{24}\cos^2\theta_{34}$	$= 4\lvert U_{e4}\rvert^2\lvert U_{s4}\rvert^2$
$\sin^2 2\theta_{\mu s}$	$= \cos^4\theta_{14}\sin^2 2\theta_{24}\cos^2\theta_{34}$	$= 4\lvert U_{\mu4}\rvert^2\lvert U_{s4}\rvert^2$

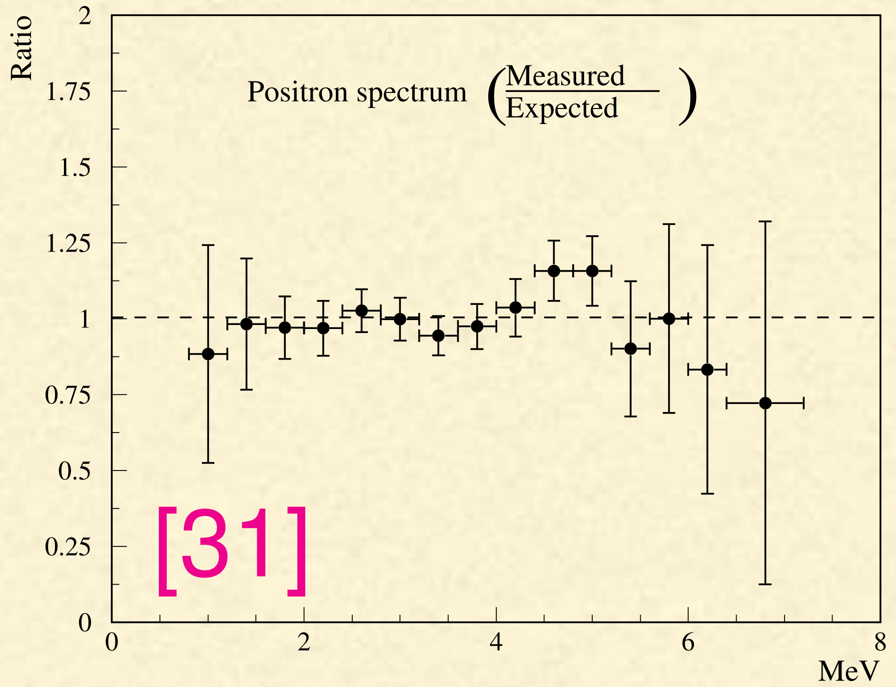
$\bar{\nu}_e$ disapp: 5 MeV excess



- Neutrino 2014: RENO [29] reported an excess of events around 5 MeV;
- seen by most reactors (also old Chooz [31]);
- DB+Prospect [30]: affect both $^{235}\mathrm{U}$ & $^{239}\mathrm{Pu}$;
- excess (not deficit) & independent of $L \Rightarrow$ **flux feature**, not ~~**sterile oscillations**~~;
- accounted by HKSS, but not by EF and KI $\Rightarrow$ reactor fluxes require further scrutiny.



⇝ [Sonzogni]



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[29] S.H Seo [RENO], talk at Neutrino 2014, Boston, USA, June 2-7, 2014

[30] F.P. An *et al.* [DB+Prospect], PRL **128** (2022) 081801 [`arXiv:2106.12251`]

[31] M. Apollonio *et al.* [Chooz], PLB **466** (1999) 415 [`hep-ex/9907037`]

sis. We note that X^k_{ij} and $\bar{X}^k_{ij}$ are independent Yukawa matrices. The fermion masses receive contributions only from X^k_{ij}, since in the Higgs basis only ϕ_h acquires a non-zero VEV while $\langle\phi_H\rangle = 0 = \langle\phi_{h'}\rangle$, leading to $X^k = \mathcal{M}_k/v$, where $\mathcal{M}_k$ are the fermion mass matrices. In this basis, $\bar{X}^k_{ij}$ are free parameters and non-diagonal matrices. Hereafter, we work in a basis in which the fermion (leptons and quarks) mass matrices are real and diagonal, where $U_k\mathcal{M}_kV_k^\dagger = m_k^{\text{diag}}$ are their bi-unitary transformations.

After rotation, one finds the following coupling strengths of the scalars h, h' and H with fermions (leptons and quarks), respectively:

$$y_f^h = \frac{m_f}{v},\; y_f^{h'} = y^f Z^{\mathcal{H}}_{32} = y^f\, s_\delta,\; y_f^H = y^f Z^{\mathcal{H}}_{22} = y^f\, c_\delta, \quad (15)$$